Modeling and Measurement of One-Phase Mixture Using Microwave Frequencies

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Abstract

In this research, the rigorous separation of a variable was used to solve the problem of scattering in near-field from a tube containing a mixture. The mixture to be used is to simulate the actual cases that occur frequently in chemical and petroleum engineering. From which, a novel technique was developed to measure one-phase of a mixture using microwave signals. Where the one-phase mixture is the volume fraction occupied by one of the phases.

The theoretical study has taken into consideration the case when the effective permittivity of the mixture is anisotropic. Theoretical and practical results have shown good agreement for the reflected and transmitted power. The results obtained by these results confirmed the possible utilization of this method for controlling and monitoring the value of mixture phases.

Keywords: One-Phase Mixture, Microwave, X-Band, Dielectric Properties, Scattering.

Introduction:

In order to get a better understanding of the interaction between the particles and molecules with microwave signals, the following concepts are necessary to know: Rayleigh-Jeans approximation, absorption, emission, and scattering of radiation. Microwave signals are very well-known in the field of communications, and became a very useful tool in several applications; such as measuring permittivity of liquid mixtures, measuring water content in a substance, sensing moisture levels [1, 2, 3, 4]. Other techniques are available for measuring the mixture [5].

Microwave signals had been used as a measurement tool in free space techniques to determine the moisture content by measuring attenuation and phase shift at X-band frequencies [6].

Microwave radiation transmissivity had been used to measure the liquid fraction of water by passing the liquid mixture through a device which measures the dielectric constant [7].

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Scattering from a dielectric cylinder has been studied by a number of researchers beginning from Lord Rayleigh [8]. However, the derivation of scattering coefficient for two coaxial dielectric cylinders has not been studied in detail, therefore in this research the complete scattering coefficient is derived, followed by the calculation of the reflected and transmitted powers, and then the case of anisotropic cylinder is also treated in detail.

Theoretical analysis:

Dielectric properties of a mixture:

A mixture of substances is defined to be a medium consisting of several constituents, each of which occupies an appreciable fractional volume of the composite medium [9]. When the mixture contains one type of inclusions, the mixture is called a two-phase mixture. In the case when the inclusions are isotropic, identical in size, shape, and orientation relative to the direction of the applied electrical field, dispersed randomly within an isotropic host medium, the effective permittivity of the mixture is anisotropic, and it is dyadic. De-Loor [10] has obtained the following expression of a two-phase mixture containing ordered ellipsoidal particle inclusions. Particles are assumed to be randomly dispersed within the host, and for convenience, their orientation is chosen such that, their semi-axes a, b, and c are along the x, y, and z-axes of a rectangular coordinate system respectively.

\[
\varepsilon_m = \varepsilon_h + \frac{\nu_i (\varepsilon_i - \varepsilon_h)}{1 + A_n (\frac{\varepsilon_i}{\varepsilon_h} - 1)}
\]  

(1)

Where:

\( \varepsilon_m \) is the average value of the dielectric constant of the mixture, \( \varepsilon_h, \varepsilon_i \) is the average dielectric constant of the host and inclusion materials respectively, \( A_n \) is the depolarization of the ellipsoid along its u-axis (u = a, b, or c), \( \nu_i \) is the inclusion volume fraction, \( \varepsilon^* \) is the effective dielectrical constant of the region immediately surrounding an included particle. For small values of inclusion volume fraction \( \nu_i \leq 0.1 \) De-Loore assigned \( \varepsilon^* = \varepsilon_h \) while for moderate to high values of \( \nu_i \) De-Loore assigned \( \varepsilon^* = \varepsilon_m \). Figure (1) shows the effective permittivity of water-air mixture versus water hold-up at 10 GHz frequency [10].

Scattering from a Dielectric Tube:

Consider a plane wave of unit amplitude incident normally on a dielectric cylinder as shown in figure (2). The direction of incidence is along the x-axis, and it's
polarization in the z-direction. The electric field in each region (after expansion in terms of Bessel functions [11]) using cylindrical coordinates is given by:

In region 1:

$$E_z = \sum_{n=0}^{\infty} \delta_n \beta_n \left( J_n(\beta_n r) + A_n H_n^{(2)}(\beta_n r) \right) \cos(n\phi)$$  \hspace{1cm} (2)

In region 2:

$$E_{2z} = \sum_{n=0}^{\infty} \delta_n \left( B_n J_n(\beta_2 r) + C_n Y_n(\beta_2 r) \right) \cos(n\phi)$$  \hspace{1cm} (3)

In region 3:

$$E_{3z} = \sum_{n=0}^{\infty} \delta_n D_n J_n(\beta_3 r) \cos(n\phi)$$  \hspace{1cm} (4)

The $J_n$ and $Y_n$ are the Bessel functions of the first and the second kind, respectively. The $H_n^{(2)}$ is the Hankel function of the second kind, and the $\beta_n$'s are the wave numbers appropriate to the various regions ($\beta_n = \alpha(\mu_n) \omega^{(2)}$), where the subscript $n$ is the medium number, and $\epsilon_n$ is the relative permittivity of the medium. $\delta_n$ is the Neumann factor (equals 1 when $n=0$, and equals 2 when $n \neq 0$). The terms $B_n, C_n$ and $D_n$ are constants. $A_n$ is the scattering coefficient.

From Maxwell equations, the scattered magnetic fields are:

$$H_{sr} = \frac{1}{j \omega \mu_0 r} \frac{\partial E_z}{\partial r}$$  \hspace{1cm} (5)

$$H_{s\phi} = \frac{-1}{j \omega \mu_0} \frac{\partial E_z}{\partial \phi}$$  \hspace{1cm} (6)

Where $\mu_0$ is the free space magnetic permeability.

The scattering coefficients are calculated according to the boundary condition that the tangential component of the electric and normal component of the magnetic fields must be continuous across the boundary, in the absence of any surface currents [10].

Hence the scattering coefficient is given by:
\[ A_n = \frac{\beta_1 M_n J_n^i(\beta_1 b) - \beta_2 N_n J_n(\beta_1 b)}{\beta_2 N_n H_n^{(2)}(\beta_1 b) - \beta_1 M_n H_n^{(2)}(\beta_1 b)} \]  \hspace{1cm} (7)

where

\[ M_n = J_n(\beta_2 b) - \frac{F_n}{K_n} Y_n(\beta_2 b) \]  \hspace{1cm} (8)

\[ N_n = J_n^i(\beta_2 b) - \frac{F_n}{K_n} Y_n^i(\beta_2 b) \]  \hspace{1cm} (9)

And

\[ F_n = \beta_3 J_n^i(\beta_3 a) J_n(\beta_2 a) - \beta_2 J_n^i(\beta_2 a) J_n(\beta_3 a) \]  \hspace{1cm} (10)

\[ K_n = \beta_3 J_n^i(\beta_3 a) Y_n(\beta_2 a) - \beta_2 Y_n^i(\beta_2 a) Y_n(\beta_3 a) \]  \hspace{1cm} (11)

In the case when the effective permittivity of the mixture is dyadic

\[
\begin{bmatrix}
D_x \\
D_y \\
D_z
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{11} & 0 & 0 \\
0 & \varepsilon_{22} & 0 \\
0 & 0 & \varepsilon_{33}
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
\]

Where \( D = \varepsilon E \), it is necessary to replace \( \beta_3 = k \sqrt{\varepsilon_3} \) by \( \beta_3 = k \sqrt{\varepsilon_{33}} \) in the expression of the scattering coefficients.

**Calculation of the reflected and transmitted powers:**

Consider the experimental set-up, (figure (3)). The reflected and transmitted powers are proportional to the power density flowing at normal incidence to these apertures. The reflection and transmission regions can be defined as follows:

- **Reflection region** \(-\pi/2 < \phi < \pi/2\)
- **Transmission region** \(\pi/2 < \phi < 3\pi/2\)

Hence the reflected power along x-direction and perpendicular to the aperture can be calculated as follows:

\[ r = (x^2 + y^2)^{1/2}, \quad \phi_n = \tan^{-1}(y/x) \]

and by Poynting theorem:

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\[ P_r = -0.5 \left[ E_{zz}(r, \phi_r) \left[ H_y(r, \phi_r) \cos(\phi_r) + H_y(r, \phi_r) \sin(\phi_r) \right] \right] \quad (12) \]

where \( P_r \) is the reflected power density, \( y \) is the distance that represents the width of the waveguide; \( x \) is the distance from the center of the cylinder to the aperture. \( * \) is the complex conjugate, \( a \) and \( b \) are the waveguide dimensions. Thus by varying \( y \), the power density along the aperture can be calculated as follows:

\[ P_r(\text{total}) = \frac{\alpha}{2} \int_{0}^{\alpha/2} P_r \, dy \quad (13) \]

The transmitted power can also be obtained using eq. (13) noting that the total shadow field in the transmitted power must be used. Figures (4 to 9) show the variation of the normalized reflected and transmitted power for different constrains.

Experimental system:

The microwave set up shown in figure (3) was used for practical measurements at different frequencies. The sweep oscillator type PM 7022x (with a frequency range of 8 to 12.4 GHz) has been used to deliver the power at a desired frequency to the system. The 10dB coupling, 40dB directivity directional coupler model x752A is used to measure the reflected power from the cylinder. The cylinder was placed in the Fresnel region from the aperture of the waveguide. The measurements of the received and reflected powers were carried out by using the HP model 415E SWR meter, and PM 7325x detector.

Results and discussion:

The practical results obtained in this research are confirmed by theoretical analysis. The practical measurements were done in the x-band frequencies. Meanwhile, the theoretical equations presented in this research can be used in theoretical analysis in the x-band up to 20GHz.

Figures (4\&5) show the normalized reflected and transmitted powers calculated for the waveguide WR(90), which are greater than the density delivered to the aperture without the cylinder. This is due to the focusing action of the tube. The focusing action occurs always in the shadow region for different values of \( \phi \), depending on the radius, and the permittivity of the cylinder. Figures (4\&5), indicate that the reflected power is more sensitive to the variation of the phase mixture than the transmitted power. The peaks in these figures are due to the phase transition that occurs in the illuminated shadow fields. Figures (6\&7) show the reflected and transmitted power for different phase mixture at 20GHz calculated using waveguide WR(42), based on theoretical analysis. Results in figures (6\&7) illustrate larger variations than the results in figures (4\&5). In figures (8\&9) the variation of the reflected and transmitted power for different frequencies at specific mixture phases were plotted. The value of the mixture was chosen
to be 1 for simplicity. The same procedure can be used to draw the transmitted and the reflected power versus frequencies at any other value of phase mixtures in theoretical and practical measurements.

Experimental results (figures 4, 5, 8, & 9) show that the discrepancy between theoretical and experimental results can be attributed to several factors such as spurious response of the detector and accuracy of the reading apparatus. During the measurements, a calibration of the oscillator output power was done before each reading. Finally, it is worth mentioning that the practical measurements have been done in the X-band frequencies only because of instrument limitations. On the other hand, the theoretical equations presented in this research can be used for theoretical analysis at different microwave frequencies.

Conclusion:

This research outlines a novel method in controlling and monitoring the value of a one-phase mixture by using microwave signals. The results were confirmed by both theoretical analysis and practical measurements.

From the above results we can conclude that, there are three parameters which can be varied to obtain a variation at a specific phase mixture. These are: the radius of the tube, the operating frequency, and the distance between the cylinder and the receiving apertures.

Further investigations are needed in other bands of microwave frequencies (below and above the X-band), with cautions on the effect of frequency on some of the parameters in the high part of the spectrum.

Figure (1); Water-Air mixture permittivity versus water phase.
Series 1 is the real component of permittivity and,
Series 2 is the imaginary component of permittivity

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**Figure (2); The geometry of the problem**

**Figure (3); The experimental set-up**
Figure (4): Normalized reflected power versus water phase.
Series 1 is the theoretical plot and,
Series 2 is the practical plot.

Figure (5): Normalized transmitted power versus water phase.
Series 1 is the theoretical plot,
Series 2 is the practical plot.
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Figure (6): Theoretical calculation of the normalized reflected power versus water phase at 20GHz frequency.

Figure (7): Theoretical calculation of the normalized transmitted power versus water phase for 20GHz frequency.
Figure (8): Normalized reflection power versus frequency for one water phase.
Series 1 is the theoretical plot and,
Series 2 is the practical plot.

Figure (9): Variation of transmitted power versus frequency for one water phase.
Series 1 is the theoretical plot and,
Series 2 is the practical plot.
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الملخص

في هذا البحث استخدمت طريقة فصل المتغيرات الدقيقة جدا لحل معادلات البصائر في المجال القريب من أجهزة عبئي على مزيج. وتشتمل المزيج لتحليل الحالات التي تتضمن جهاز متكرر في تطبيقات كبيرة كما في الهندسة الكيميائية والطبية وغيرها. استنادًا إلى طريقة فصل المتغيرات تم استخدام طريقة جديدة لقياس الدقة المتكررة للحالة المتزامنة باستخدام دقة متكررة. الدراسة النظرية أخذت بنظر الاعتبار الحالات التي يكون فيها المزيج ذو خواص ليست واحدة في جميع الاتجاهات. التحليلات النظرية والنتائج العملية كانت متقاربة من حيث القدرة المنخفضة والمرونة. نتائج هذا البحث أثبتت إمكانية استخدام هذه الطريقة للمراقبة وتطبيقها في أطوار المزيج.

References


