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Non-Markovian Gain in Quantum-Dot Semiconductor Lasers

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Abstract

In this work gain of quantum dot (QD) structures is studied theoretically using non-Markovian line shape function. The gain is shown to be higher due to many-body effects. In the contrary to the Markovian assumption, the absorption region is shown to be in a good coincidence with the Fermi level separation as expected. The correlation time is shown to be have a considerable effect on Markovian gain.

Keywords: Gain, Markovian gain, Non-Markovian gain, Band-gap renormalization, Line shape function.

Introduction

Quantum dot (QD) structures are the top contending nano-structure technologies in this century. The scientists hope that nano-electronic circuits are possible around 2040 [1,2].

Bulk (conventional) semiconductor laser consists of low band-gap active region sandwiched between two higher band-gap cladding layers. Reduction (quantization) in the thickness of the active region results in a quantum-well (QW) laser. This results in discrete energy levels in one dimension as a quantum potential well (mechanical) problem. Quantum-well structures introduce a good example of quantum mechanics, but they make very good lasers [3]. Quantization (reduction) of active region in two dimensions introduces quantum-wire (QWi) structures. While the reduction in three dimensions gives quantum dot (QD) structures. These three types of structures are classified as a quantum-well family, which is a part of nano-structures family also.

QW lasers introduce advantages better than conventional lasers [4] such as the ability to vary lasing wavelength merely by changing the width of the quantum-well. They can generate more power with fewer injected carriers required. They deliver more gain with less change in refractive index.

The optical gain is probably one of the most important properties of semiconductor quantum wells. It results from polarization induced by a light-wave train in an excited laser medium. Conventional theoretical calculations [5, 6] are usually based on the

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density matrix formalism with intraband relaxation time. The effect of intraband relaxation is to disturb this polarization by various scattering processes. Scattering acts as frictional resistance on oscillation of the polarization, and the resonance spectrum of the oscillation to light-wave train becomes broad. In this case, [7] the broadening of energy states below the band-gap is used to explain transitions. A Lorentzian line shape function is used to describe homogenous broadening which results from very short carrier scattering times (< 1ps). The overall gain profile and its magnitude depend considerably on the line shape function.

The detailed balance between absorption and emission of photons requires [7] that the transparency point in the gain spectra coincides with the Fermi (or quasi-Fermi) level energy. In general the gain spectra with the Lorentzian function don't satisfy this condition, and a deviation from the experimental results is pointed out.

In addition, there are many-body effects to be considered in the description of the optical gain spectra such as the reduction of band-gap with increasing carrier density (the band-gap renormalization BGR), the enhancement of optical gain due to the attractive electron-hole interaction (Coulomb or excitonic enhancement), and the plasma screening. On the other hand, recent investigations [7] of the optical response show that the decay dynamics of the polarization in semiconductor indicate strongly non-exponential decay, which can be satisfactorily described if the processes are non-Markovian. The physical nature of the dephasing is due to the interactions of the interband polarization of the excited system with its surroundings. These interactions cause the fluctuation of the transition frequency within a certain spectral distribution and the homogenous dephasing [8]. These relaxation kinetics in non-equilibrium cases are often characterized by the presence of memory effects on a very short time scale and the equation of motion for the system have time-convolution form of integral kernels. The memory effects arise because the wave functions of the particles are smeared out so that there is always some overlap of wave functions and as a result the particle retains some memory of the collisions it has experienced through its correlation with other particles in the system. The memory effects are the characteristics of the quantum kinetic equations. As a result, the strict energy conservation may no longer hold [9] for a time interval shorter than the correlation time.

QW and QWi line shape functions were analyzed taking into account non-Markovian relaxation [9]. Ahn [10] derived time convolutionless (TCL) equations for the system of interacting electron-hole pairs under arbitrary external optical field. TCL equations take into account non-Markovian relaxation and the renormalization of the memory effects. BGR, and many-body for quantum-well structures were studied thereafter [7].

Depending on the TCL equations and the results for QW lasers, the gain in the quantum-dot laser is obtained in this work. Here, the theoretical approach of the electronic properties is based on the parabolic band assumption for both conduction and valence bands [4,5]. InGaAs/InP material system is used as an example to study non-Markovian gain since this material gives (1.32 μm) wavelength which is important for optical communications. A study of line shape function and gain using both Lorentzian and non-Markovian assumptions is done. A comparison with QW and QWi structures is also done.

II. Theory

TCL equation for reduced density operator of a stochastic reservoir has been derived by Ahn in [10]. Gain in QD structures is gotten there in the energy (or frequency) domain. The information of the system is contained in the reduced density operator obtained by eliminating the dynamical variables of the reservoir from the total density operator using projection operator. The non-diagonal interband matrix element $p_k^*(t)$ that describes the interband pair amplitude induced by the optical field. Depending on the TCL equation and the results for QW lasers [10, 7], the optical gain $g(w)$ for a zero-dimensional* lasers can be given by the relation:

$$g(w)=\frac{w\mu c}{n_r}\int g_{cv}\,\mathrm{Re}\,\Xi(0,\Delta_{nml})\left|\hat{\varepsilon}.\vec{M}\right|^2.[1+\mathrm{Re}\,g_2(\infty,\Delta_{nml})][n^0_{cnml}-n^0_{vnml}]dE_{cv} \qquad \text{......... (1)}$$

Where μ is the permeability, n_r is the refractive index of the active material, c is the speed of light in free space, g_{cv} is the density of states of the QD (=2/V), and $\hat{\varepsilon}$ is a unit vector along the polarization direction of the optical field, $\vec{M}$ is the momentum matrix element. Energy levels in QD are characterized by three quantum numbers n, m, and l in the x, y, and z directions, respectively, for each conduction or valence bands. E_{cv} is the transition energy between conduction and valence bands. In eq. (1), n^0_{cnml} and n^0_{vnml} are the quasi equilibrium (Fermi) distribution functions of electrons in the conduction and valence bands, respectively. Their values can be calculated using the well known relations, for example, see [3] p.157.

$\mathrm{Re}\,\Xi(0,\Delta_{nml})$ represents the line shape function that describes the spectral shape of the optical gain in a driven semiconductor. It was shown by Ahn [10] that the line shape of the gain spectra is Gaussian for the simplest non-Markovian quantum kinetics:

$$\mathrm{Re}\,\Xi(0,\Delta_{nml})=\sqrt{\frac{\tau_c\pi}{2\gamma_{cv}}}\exp\left(-\frac{\tau_c\Delta^2_{nml}}{2\gamma_{cv}}\right) \qquad \text{... (2)}$$

where:

$$\gamma_{cv}=\frac{\hbar}{2}\left(\frac{1}{\tau_c}+\frac{1}{\tau_v}\right) \qquad \text{... (3)}$$

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* QD structures are called as zero-dimensional (0-D) structures since there are 0- dimensions unquantized. When they called three-dimensional structures one looks for the 3-dimensional quantized. 0-D is a commonly used in literature.

τ_c and τ_v are the correlation times for the intraband relaxation in the conduction and valence bands respectively, and

$$\Delta_{nml}(0) = E_{cnml} - E_{vnml} + E_g - \hbar w \qquad \text{.................................. (4).}$$

In the case of enhancement due to the excitonic effects (caused by the Coulomb interaction) the line shape function become describes by [7] the function $[\mathrm{Re}\,\Xi(0,\Delta_{nml})/(1-\mathrm{Re}\,q_{1nml}(0))]$ instead of $\mathrm{Re}\,\Xi(0,\Delta_{nml})$. Note that $\mathrm{Re}\,q_{1nml}(0) = \sum_{nml} \mathrm{Re}\,\Xi(0,\Delta_{nml})[n^0_{cnml} - n^0_{vnml}]$.

The factor $(1+\mathrm{Re}\,g_2(\infty,\Delta_{nml}))$ in eq. (1) describes the gain (or line shape) enhancement due to the interaction between the optical field and the stochastic reservoir of the system. Where $g_2(\infty,\Delta_{nml})$ represents the interference term between the optical field and the reservoir. It describes the band-gap renormalization. Note that [12]:

$$\mathrm{Re}\,g_2(\infty,\Delta_{nml}) = \frac{\gamma_{cv}\tau_c}{1+\Delta^2_{nml}\tau^2_c} \qquad \text{...(5)}$$

which vanishes in the Markovian limit (also in the case of non Markovian without renormalization).

Note that E_{cnml} and E_{vnml} are the energy levels in the conduction and valence bands. As referred earlier the parabolic band model is used in this work to calculate these levels. This is done by solving the Schrödinger equation numerically which can be found in many works, for example, see [4, 5, 6].

III. Results and Discussions

(10x10x10) nm^3 of InGaAs/InP quantum dot laser is used to study non-Markovian effect on gain. Band-gap renormalization [12] is used through the calculations. The correlation times are $\tau_c = 0.2ps$, and $\tau_v = 0.07ps$ unless stated else ware. The intraband relaxation time is $\tau_{in} = 0.1ps$.

Figure (1) shows the QD gain spectra for three cases: 1) Markovian model with Lorentzian line shape, 2) non-Markovian model, and 3) non-

Markovian model with renormalized memory effects $(1+\mathrm{Re}\,g_2(\infty,\Delta_k))$ is included.

Figure (2) shows the non-Markovian line shape function for two correlation times ($\tau_c = 0.1ps$ and $\tau_c = 0.2ps$), Lorentzian line shape function is also included for comparison. It is obvious that the longer correlation time the higher line shape function. Maximum full width at half maximum is seen when $\tau_c = 0.1ps$. Since the tail of the non-Markovian line shape decreases much faster than that of the Lorentzian, the gain magnitude of the Lorentzian line shape function may be underestimated as seen in Fig.(1).

Figure (3) shows gain spectra for non-Markovian assumption with (and without) renormalized memory effects $(1+\operatorname{Re} g_2(\infty,\Delta_{nml}))$. The Markovian gain is also seen. These spectra are drawn at 3×10^{18} cm^{-3} carrier density. The Fermi level separation at this carrier density equals (1.001 eV) for QD structure. As mentioned earlier, the Markovian gain with Lorentzian line shape function produces an anomalous absorption region below the band-gap energy.

In the absence of spectral broadening, the optical gain spectra are related to the spontaneous emission spectra from the detailed balance between absorption and emission of photons [7,13]. One can easily see from this relation that there is a transparency point in the gain spectra that coincide with the Fermi (or quasi-Fermi) level separation. This suggests that carriers and photons are in thermal (or quasi) equilibrium. Fig. (3) shows two anomalies for Lorentzian function: unnatural absorption region below the band-gap energy and mismatch of the transparency point in the gain spectra with Fermi level separation, the latter suggests that the carriers and the photons are not in thermal (or quasi) equilibrium. On the other hand the non-Markovian gain spectra with or without many body effects do not have an anomalous absorption below the band-gap and the discrepancies between the transparency points and the Fermi level separation are much smaller than those of the Markovian model.

Figure (4) shows a comparison between non-Markovian gain spectra of quantized structures. QD maximum gain is three times greater than that of QWi, and more than ten times than that of QW. A comparison between Lorentzian and non-Markovian gain spectra for both QW and QWi is shown in Fig. (5). The increment ratio is seen in Table (1). It is obvious that the increment ratio is high for higher quantized structures. A noted point can one notice from figures (3-5) that the QD gain spectra are more “isotropic” on both sides of the maximum gain point than QW and QWi curves. This can be reasoned to complete quantization of QD structures.

Conclusions:

Gain and line shape function in QD is studied using non-Markovian assumption, the results are compared with those under Markovian assumption. The following points can be assigned as conclusions:

1. Gain values of Markovian assumption are under estimation due to neglecting effects such as many-body effects which are taken into account in non-Markovian assumption.
2. Non-Markovian assumption removes unnatural transparency point in absorption spectra as a result of taking most dephasing effects into account.
3. QD spectra are higher and more “symmetric” than other structures due to complete quantization of energy levels.

خوارزميات موسعة للتدرج المترافق في ربط نماذج غير تربيعية في البرمجة اللاخطية

امين الخرسان، حميد عبود، فلاح حنون

ملخص

في هذا البحث، تمت دراسة خواص الكسب في ليزر شبه الموصل النقطي الكمي نظرياً باستخدام دالة شكل الخط من النوع الجاوسي (استرخاء الحزم غير الماركوفاني) ثم حساب مستويات الطاقة لهذا الليزر مع إفتراض كون الحزم بشكل قطع مكافيء. لد وجد إن التحصيل يزداد نتيجة تأثيرات الجسيمات المتعددة. لقد أظهر التقريب الماركوفاني وجود منطقة امتصاص غير طبيعية في طيف الكسب حيث إنها لا تنطبق مع مقدار طاقة الفصل لمستويي فيرمي في حزمتي التوصيل والتكافؤ لتركيب الليزر شبه الموصل المستخدم. هذا التصرف يتناقض مع افتراض التوازن الحراري للحاملات والفوتونات. وأن شكل الخط من النوع غير الماركوفاني تجعل منطقة الامتصاص متطابقة مع طاقة الفصل لمستويي فيرمي. أجريت كذلك المقارنة مع الليزرات الكمية الأخرى (الشريطي والسلكي).

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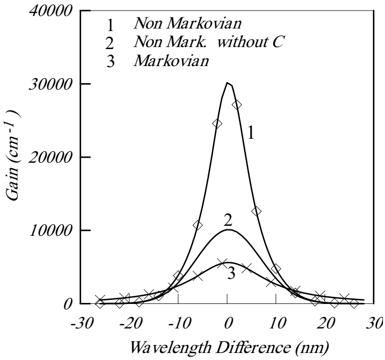

Fig.(1): QD Gain spectra for 1) non-Markovian relaxation with renormalized memory effects, 2) non-Markovian relaxation, and 3) Markovian relaxation.

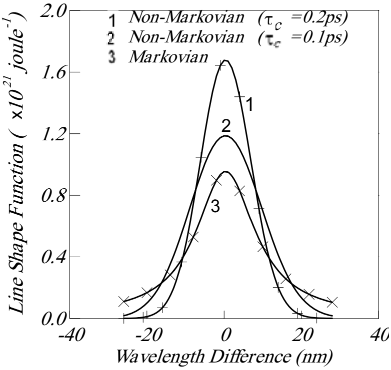

Fig.(2): QD Line shape function for : 1)Non-Markovian function with $\tau_c = 2ps$, 2) Non-Markovian function with $\tau_c = 1ps$, and 3) Markovian function.

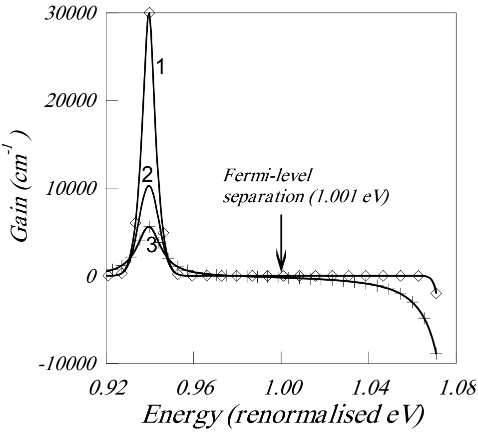

Fig.(3): QD gain spectra for the following assumptions: 1) Non-Markovian with renormalized memory effects, 2) Non-Markovian, and 3) Markovian. The correlation time $\tau_c = 2ps$.

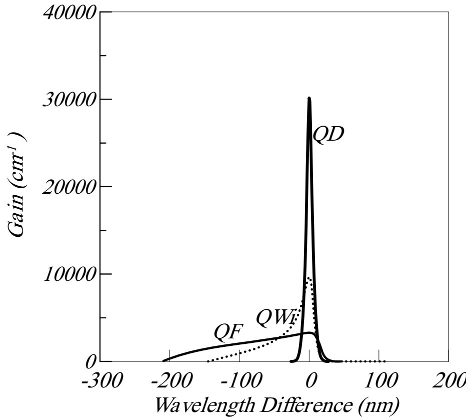

Fig. (4): A comparison between QW gain spectra: 1)QD, 2) QWi, and 3)QW.

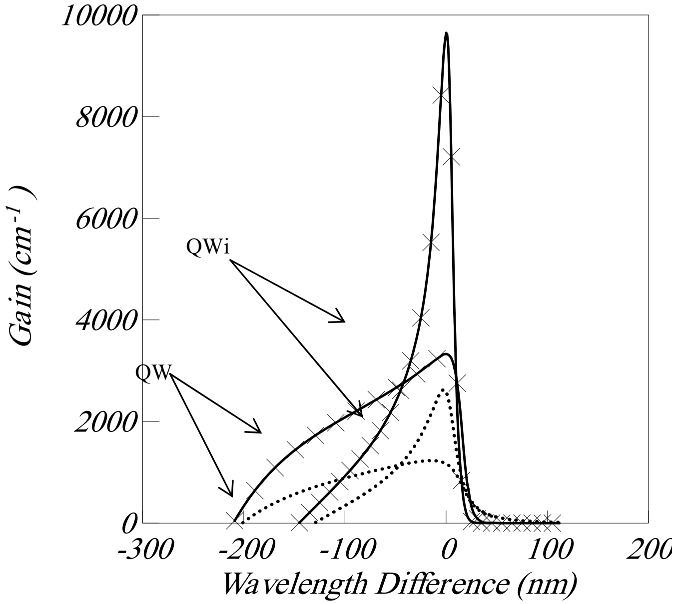

Fig.(5): Gain spectra for quantized (QW and QWi) structures, with Markovian (black curves) and non-Markovian (black curves with crosses) functions.

Table (1): Lorentzian and non-Markovian gain values, and Increment ratio for quantized structures (QD, QWi, and QF)

Quantized structure	**Lorentzian gain (cm^{-1})**	**Non-Markovian gain (cm^{-1})**	**Increment ratio per carrier number**
QD	5601	30161	1.79
QWi	2624	9694	4×10^{-5}
QW	1229	3326	3×10^{-6}