Yarmouk University
Faculty of Science
Department of Mathematics

Edge Maximal Graphs Without Some Specific Graphs

by
Hanan Saleh Abushahma

Supervisor
Prof. Ahmad Al-Rhayyel

April 3, 2013
Edge Maximal Graphs Without Some Specific Graphs

by

Hanan Saleh Abushahma

Supervisor

Prof. Ahmad Al-Rhayyel

April 3, 2013
Edge Maximal Graphs Without Some Specific Graphs

By
Hanan Saleh Abushahma
B.Sc., Mathematics, Yarmouk University, 2013.

Submitted in partial fulfillment of the academic requirements for the degree of Master of Science in the Department of Mathematics, Yarmouk University, Irbid, Jordan.

Supervisor
Prof. Ahmad A. Al-Rhayyel
Professor of Mathematics, Yarmouk University.

April 3, 2013
بسم الله الرحمن الرحيم

آمِنَّا يَا سيِدَ الْذَّي خَلَقَ ۖ خَلَقَ الْإِنسَانَ مِنْ عَلْقٍ. ۖ أَقِمْ وَبِكَ الْأَكْرَمَ. ۖ الْذَّي عَلَّ رَبَّكَ بِالْقَيْدِ. ۖ عَلَّ الْإِنسَانَ مَا أَرَيْنَهُ. ۖ صَدِقَ اللَّهُ العَظِيمِ.
Edge Maximal Graphs Without Some Specific Graphs

By

Hanan Saleh Abushahma

B.Sc., Mathematics, Yarmouk University, 2013.

Submitted in partial fulfillment of the academic requirements for the degree of Master of Science in the Department of Mathematics, Yarmouk University, Irbid, Jordan.

Approved by:

Prof. Ahmad Al-Rhayel .............. Chairman
Professor of Mathematics, Yarmouk University.

Dr. Mohamad Bataineh ..................... Member
Assistant Professor of Mathematics, Yarmouk University.

Prof. Khaled Al-Sharo ......................... Member
Professor of Mathematics, Al-Abayt University.

2013
Dedication

To my parents, my husband and my daughters.
Acknowledgement

In the name of Allah, Most Gracious, Most Merciful.

I would like to take this opportunity to convey my sincere thanks and deepest gratitude to my supervisor prof. Ahmad Al-Rhayyel.

I am very grateful to him for all the help and invaluable guidance, fruitful discussions, patience and continued encouragement provided to me at every stage of this thesis.

Not forgetting to convey my sincere thanks to my committee members Dr. Mohammed Bataineh, prof. Khaled Al-Sharo.

Finally, I would like to thank my family for their love, and support during my study.
Table of contents

Acknowledgement ........................................................... I
List of contents..................................................................... II
Abstract................................................................................ III
Arabic Abstract........................................................................ IV
Introduction.............................................................................. 1

Chapter One. Preliminaries of graph theory
1.1 Basic definitions and notations................................. 3

Chapter Two. Review of known results of
Turán’s-type extremal graphs problems
2.1 Turán’s-type extremal problems............................... 11
2.2 Recent results on Turán’s extremal problems.. 19

Chapter Three. Edge maximal graphs without W_7
3.1 Preliminaries............................................................... 24
3.2 Edge maximal graphs without W_7....................... 25

References............................................................................. 69
Abstract

Let $G(n;W_k)$ denote the class of graphs on $n$-vertices having no $W_k$-graph and

$$f(n;W_k) = \max\{\varepsilon(G); G \in \mathcal{G}(n;W_k)\}.$$ 

In this thesis we determine $f(n;W_7)$ by proving that:

$$f(n;W_7) \leq \begin{cases} 17 & \text{if } n = 7 \\ \left\lfloor \frac{n^2}{4} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil + 2 & \text{if } n = 8,9,10,11,12,13. \end{cases}$$

Key words: Wheel graph, Extremal graph.
ملخص

افرض أن $G(n; W_k)$ يرمز لصف من البيانات على $n$ من الرؤوس، والتي لا تحتوي على البيان $W_k$.

$$f(n; W_k) = \max\{\epsilon(G); G \in G(n; W_k)\}.$$ 

في هذه الأطروحة نحدد $f(n; W_7)$ ببرهان أن:

$$f(n; W_7) \leq \begin{cases} 
17 & \text{if } n = 7 \\
\left[\frac{n^2}{4}\right] + \left[\frac{n}{2}\right] + 2 & \text{if } n = 8, 9, 10, 11, 12, 13.
\end{cases}$$

الكلمات المفتاحية: بيان العجلة، البيان الأعظمي.
Introduction

Graph theory is a branch of mathematics which has many applications into other sciences such as chemistry, communication networks, traffic networks, engineering, computer science and other sciences.

Some branches of graph theory are concerned with establishing the best bounds for graph parameters and characterizing the graphs for which the bounds are achieved. Such theory is known as extremal graph theory ([12], [13],[14]) which forms the main purpose of this thesis.

Extremal graph theory is concerned with the relationships between the parameters such as: number of vertices, number of edges, girth, circumference, etc.

In this thesis we mainly focus on studying a particular problem in extremal graph theory, the so called Turán’s-Type Extremal Problem. This kind of problem has become a frequently investigated research topic since Turán (1941) published his famous result. In this thesis we consider the problem concerning graphs that do not contain the wheel graph $W_7$.

In chapter one we introduce some of the basic definitions and notations in
graph theory, in chapter two we survey some results concerning extremal type problems and finally in chapter three we consider the Turán’s-Type Extremal Problem with the wheel graph being the forbidden subgraph, and we determine \( f(n; W_7) \) by proving that:

\[
f(n; W_7) \leq \begin{cases} 
17 & \text{if } n=7 \\
\left\lfloor \frac{n^2}{4} \right\rfloor + \left\lfloor \frac{n}{2} \right\rfloor + 2 & \text{if } n=8,9,10,11,12,13
\end{cases}
\]