Effect of Residual Stresses on Elastic-Plastic Fracture Mechanics of Ductile Cast Iron Pipe under Pressure Loading

Haider Jasim* and Haithem Ali **

Received on March 2, 2009 Accepted for publication on June 24, 2009

Abstract

In this paper, an investigation was performed to examine the influence of the residual stress on the elastic-plastic fracture mechanics (EPFM) of ductile cast iron pipe (DCIP) having radial crack using analytical and finite element method (FEM). The neutron diffraction technique was used to measure the residual stresses. Crack initiation and growth behavior of DCIP was assessed using SINTAP–Procedure by drawing failure assessment diagram. As a result the residual stresses accelerated the radial crack growth in DCIP and increase the stress intensity factor values. For DCIP, there will be significant unstable crack growth before final fracture.

Keywords: Residual stresses; Stress intensity factor; Failure assessment diagram; FEM; Radial crack; Elastic-plastic fracture mechanics.

Nomenclature

- $R_1$: Inner pipe radius.
- $R_2$: Outer pipe radius.
- $t$: Pipe wall thickness.
- $\sigma_y$: Yield strength.
- $\sigma_t$: Tensile strength.
- $B$: Strain-displacement matrix.
- $\sigma$: Stress vector.
- $F_y$: Pressure force vector.
- $N_i(x, y)$: Element shape function.
- $n$: Number of nodes.
- $K_e$: Element stiffness matrix.
- $P_e$: Element force vector.
- $n_e$: Number of elements.
- $[U]$: Global displacement vector.
- $J_{\omega}, J_{\omega}$: $J$-values for material.
- $J_x, J_y$: Jacobian parameter in x and y directions.
- $P$: Pressure at interface between plastic and elastic zone.
- $K_T$: Total stress intensity factor under mode I conditions.
- $A$: Parameter.
- $D_e$: Elastic-plastic D-matrix.
- $\sigma, \sigma, \phi$: Mean and invariant stresses.
- $H$: Yielding surface.
- $\lambda$: Undetermined constant.
- $R$: Force vector.
- $U_i$: Unknown displacement vector.
- $U_i$: Prescribed displacement vector.
- $P$: Unknown force vector.
- $P$: Prescribed force vector.
- FAC: Failure assessment curve.

© 2009 by Yarmouk University, Irbid, Jordan.

* Chemical Eng. Dept., College of Eng., Basrah University, Basrah, Iraq.
** Staff Eng. at laboratory of petroleum pipe company, Basrah, Iraq.
Introduction

Cast iron is classified as grey and white. In 1955 the ductile cast iron was introduced, when alloying elements were added to make iron less brittle. It was also called nodular cast iron since the shape of the graphite inclusion in the iron is changed to a nodular form. The nodular shape of the graphite inclusions decreases the risk of getting cracks in the material. In general ductile iron is the most commonly used material for water pipes and petroleum pipelines since it is economical to produce and is able to withstand the larger internal pressure.

Most ductile cast iron pipes (DCIP) are manufactured in large dimensions because they are less complicated to manufacture and are less expensive. DCIP can be used aboveground or underground applications. The failure of DCIP is usually determined by the iteration between the defects with DCIP and the stresses to which it is exposed. The stresses are combination of those applied in service and those which remain in material in the absence of any external forces, namely residual stresses [1].

The nature of residual stresses in DCIP can arise by two ways: first, residual stress remain in DCIP after manufacturing and processing treatment (hot and cold working process). Second, residual stress induced in practical application, i.e., applying the external loading (pressure or thermal loading) that was sufficient to yield the pipe and localized plastic region, if the load is removed elastic occurs and leaving the residual stresses in material.


In this paper, a 2D elastic-plastic FEM and SINTAP (Structural Integrity Assessment Procedure for European Industry) Procedure employing residual stress are applied to evaluate the structural integrity of DCIP by drawing failure assessment diagram. The results obtained are compared with those obtained from experiment using neutron diffraction technique. As a result the residual stresses accelerated the radial crack growth in DCIP and increased the stress intensity factor values.

DCIP Properties and Dimensions

Fig.1 shows a ductile iron pipe photograph used for transmission of petrol, has the following dimensions and elastic properties [5]:

- Normally length = 216'' (5.48 m)
- \( R_1 = 19.15'' \) (486.41 mm)
- \( R_2 = 18.64'' \) (473.456 mm)
- \( t = 0.51'' \) (12.954 mm)
Effect of Residual Stresses on Elastic-Plastic Fracture Mechanics of Ductile Cast Iron Pipe under Pressure Loading

\[ E = 169 \text{GN/m}^2, \sigma_u = 414 \text{MN/m}^2 \]
\[ \sigma_y = 276 \text{MN/m}^2, \rho = 7100 \text{kg/m}^3 \]
\[ \nu = 0.29, \quad K_{IC} \geq 50 \text{MPa} \sqrt{\text{m}} \]

**Residual Stresses Analysis**

Residual stresses may be determined by three methods: analytical (i.e., the local strain approach), computational with finite element method, and experimental (the most common used).

Fig.2, shows DCIP having inner radius \( R_1 \) and outer radius \( R_2 \), subjected to an internal pressure \( P_i = 80 \text{MPa} \), to produce yielding of radius \( R_y \). The radial elastic stress \( (\sigma_{Re}) \) and hoop elastic stresses \( (\sigma_{He}) \) in elastic zone as functions of the radial position according to [6]:

\[
\sigma_{Re} = \frac{P_i R_2}{R_2^2 - R_1^2} \left[ \frac{1}{1 + \frac{R_2^2}{R_1^2}} \right] \quad (1)
\]

\[
\sigma_{He} = \frac{P_i R_2}{R_2^2 - R_1^2} \left[ 1 - \frac{R_2^2}{R_1^2} \right] \quad (2)
\]

And the plastic pressure is:

\[
P_p = \frac{\sigma_y}{2R_2^2} \left( R_2^2 - R_y^2 \right) \quad (3)
\]
The plastic radial and hoop stresses in plastic zone were calculated using the following equations [6]:

\[
\sigma_{rp} = \sigma_y \left[ \ln \frac{R}{R_p} - 0.5 \left( 1 - \frac{R_p^2}{R^2} \right) \right]
\]

\[
\sigma_{hp} = \sigma_y \left[ 1 + \ln \frac{R}{R_p} - 0.5 \left( 1 - \frac{R_p^2}{R^2} \right) \right]
\]

According to pressure technology association code of practice the plastic zone extends to the geometrical mean radius \( \sqrt{R_1 R_2} \) [7].

Neutron Diffraction Technique

Neutrons subatomic particles found as part of the nucleus of almost all atoms. The helium atom has two neutrons and two protons in its nucleus; the carbon atom nucleus has six neutrons and six protons, while the iron atom has 36 neutrons and 20 protons [8].

Since neutrons penetrate readily into most materials, neutron diffraction has become a well established method for non-distractively measuring residual stresses in engineering component. This technique sends a beam of neutrons into a crystalline material. Fig.3 illustrates the basic principle of diffraction from single set of lattice plane. In neutron diffraction method a neutron beam of constant wavelength of the incident beam \( \lambda \) is diffracted through a scattering angle \( 2\theta \). The spacing d between atomic planes in the lattice is then calculated according to Bragg’s law [9]:

\[
\lambda = 2d \sin \theta
\]
Effect of Residual Stresses on Elastic-Plastic Fracture Mechanics of Ductile Cast Iron Pipe under Pressure Loading

The lattice residual strain is a functional change on the lattice spacing with reference to the stress free lattice spacing $d_0$, i.e.,

$$\varepsilon = \frac{d - d_0}{d_0}$$  \hspace{1cm} (7)

The measurement arrangement shown in Fig. 4 is used to take radial and circumferential residual strain.

**Figure 3**: Neutron diffraction.

**Figure 4**: Measurement of residual stresses using neutron diffraction method.
**SINTAP-Procedure**

The SINTAP (structural integrity assessment procedure for European industry) Procedure offers a good approach to draw failure assessment diagram FAD, which is given as follows [10]:

The basic equation of SINTAP-FAD route:

\[ K_r = f(L_r) \]  \hspace{1cm} (8)

With \( K_r \) being \( \frac{K_T}{K_{mat}} \) and \( L_r \) being the applied load normalized by the limit load.

The function \( f(L_r) \) is given as:

\[ f(L_r) = \left[ 1 + 0.5L_r^2 \right]^{0.5} \text{ for } 0 \leq L_r < 1 \]

And

\[ f(L_{r,1}) = \left[ \frac{1}{\lambda} + \frac{1}{2\lambda} \right]^{-0.5} \text{ for } L_r = 1, \]

where,

\[ \lambda = 1 + \frac{E\Delta \varepsilon}{\sigma_y} \]

And

\[ f(L_r) = f(L_{r,1}) \times L_r^{(n-1)/2n} \text{ for } 1 \leq L_r < L_r^{\text{max}}. \]

The Luders strain is estimated from an empirical correlation:

\[ \Delta \varepsilon = 0.0375 \left[ 1 - \frac{\sigma_y}{1000} \right] \] \hspace{1cm} (9)

The strain-hardening exponent is determined on conservative empirical basis:

\[ n = 0.3 \left[ 1 - \frac{\sigma_y}{\sigma_r} \right] \]
The plastic collapse load is defined as:

\[ L^\text{max}_p = \frac{1}{2} \left[ \frac{\sigma_x + \sigma_y}{\sigma_y} \right] \]

The initial values of J-applied (J_{i-material}) can be calculated from the following relation:

\[ J_i = J_{i\text{-mat}} \left[ f(L_p) \right]^{-2} \quad (10) \]

**Stress Intensity Factor and Residual Stress**

The presence of crack in DCP reduces the strength because the stresses and strains are highly magnified at the crack tip. The use of parameter to describe the local stresses and strains magnification at crack tip is important to evaluate structural integrity. This parameter is called stress intensity factor \( K_I \). \( K_I \) defines the stress field near crack tip and provides fundamental information on how the crack is going to propagate. Residual stresses influence on the values of \( K_I \). For linear elastic fracture mechanics \( K_I \) can be calculated without residual stresses effect, and then for elastic-plastic analysis the effect of residual stresses can be employed, i.e., \( K_{\text{res}} = K_I \) can be calculated under pressure loading conditions [8].

The finite element method (FEM) simulation of a preloaded DCP was chosen as a test case with the accuracy of various methods to determine the stress intensity factor with residual and without residual stress. Fig.5 shows finite element meshes by using 9-nodes two dimensional isoperimetric element. The total number of elements is 400 elements with 1194 nodes was used for analysis.

The value of J-Integral in elastic portion for a pipe having radial crack and for plain strain condition is given by [11]:

\[ J_e = \frac{1 - \nu^2}{E} K_I^2 \quad (11) \]

The elastic-plastic J-Integral equation for two dimensional problems is:

\[ J = \int f \left[ W dy - T \frac{\partial u}{\partial x} ds \right] \quad (12) \]

where,

\[ W = W_e + W_p \]
Appendix contains procedure used in elastic-plastic analysis which is used to determine the residual stresses.

Results and Discussion

Fig.6 and Fig.7 show the tangential and radial residual stresses distribution calculated using analytical method, finite element method and neutron diffraction method. The results are plotted against radius. As illustrated the data obtained from analytical and FEM is a good agreement with the experiment data. The residual stresses have the same distribution in the two cases simulation and experiment. The tangential residual stresses show compressive and tensile stress region, the minimum at inner radius, and the increase through thickness. It is noted that the tensile radial residual stress is larger than the compressive one. The transition of compression to tension for radial and hoop residual stresses occurs at the same region.

Fig.8 shows failure assessment diagram (FAD) and loading curve. The (FAD) getting from drawing the values of \((K_r)\), i.e., the stress intensity factor divided by fracture toughness of material and \((L_c)\), i.e., applied load divided by limit load. From this figure, two regions can be seen: stable region (safe) which is defined as the conditions of crack growth during which the applied value of \((K)\) for linear elastic conditions equals the resistance of material to fracture; and unstable region (unsafe), which shows fracture condition to propagate until complete fracture occurs.
The point where intersects between failure assessment diagram and loading curve give the value of \( L_r \) for crack initiation and from this crack initiation plastic pressure can be calculated from relation \( L_r \times P_p \). In Fig. 6, \( L_r = 0.24 \) without residual stresses and the crack which initiate plastic pressure \( 3675 \times 0.24 = 882.5\, kPa \), and \( L_r = 0.22 \) when taking residual stress and crack initiate plastic pressure is \( 3675 \times 0.22 = 808.5\, kPa \). From this we can see that the residual stress will reduce the plastic pressure which cause to crack initiates.

Fig. 9 shows the J-values in case elastic-plastic is obtained from FEM and SINTAP-Procedure with and without residual stress. It is clear that the residual stresses will raise the values of J-values resulted from increasing stresses. Small difference may appear due to different types of calculation.

**Figure 6:** Tangential residual stress distribution.
Figure 7: Radial residual stress distribution.

Figure 8: Failure assessment diagram
Conclusions

Fracture mechanics is a useful tool for assessing the integrity of DCIP having crack. From the discussion above we can conclude the following remarks:

1- For the first time in the literature, cross sectional measurement were made of the residual stresses in DCIP subjected to pressure loading and the measurement compared with analytical and finite element model.

2- An increase in the crack length will cause to increase in the J-values, results from increasing stresses on crack tip.

3- The fracture of DCIP in general is accompanied by a large amount of plasticity result from instability crack extension during ductile fracture.

4- The relation between J-values and $\frac{\sigma}{\sigma_y}$ is not linear.

5- The residual stress distribution in DCIP is non-linear and not uniform.

Acknowledgement

The author expresses all thanks to staff in laboratory of petroleum pipe company, Basrah, for their help and support for testing pipes, and for useful notes and advices during writing this paper.
تأثیر الإجهاد المتبقية على ميكانيكية الانكسار المرن للأنابيب حديد الزهر المطاط تحت تأثير أحمال الضغط

جعير هادي جاسم و هيثم عباس علي

ملخص

في هذا البحث، أُجريت دراسة على طريقة العناصر المحددة والطرق التحليلية تم تحليل ودراسة تأثير الإجهاد المتبقية. في ميكانيكية الانكسار المرن للأنابيب حديد الزهر المطاطية التي تمتلك شقوقة القطرية. استخدمت طريقة انسحاب النيترونين للقياس الإجهاد المتبقية المتغير مع طريقة العناصر المحددة والطريق التحليلية. طبقت طريقة SINTAP المتضمن الإجهاد المتبقية لتقييم التركيب الكامل للأنابيب حديد الزهر المطاطية وكذلك بداية نمو ثقوب عن طريق رسم مخطط تقييم الفشل. وجدت النتائج أن الإجهاد المتبقية تسبب نمو ثقوب وتزيد من قيمة معدل شدة الإجهاد. نمو الثقوب يكون بصورة غير مستقرة قبل حصول الانكسار النهائي.

الكلمات الدالة: الإجهاد المتبقية، معامل ثدة الإجهاد، مخطط تقييم الفشل، طريقة العناصر المحددة، الشقوقة القطرية، ميكانيكية الانكسار المرن للأنابيب.

References


Effect of Residual Stresses on Elastic-Plastic Fracture Mechanics of Ductile Cast Iron Pipe under Pressure Loading


Appendix

The procedure used can be simulated by using displacement method as follows [12, 13]:

**Step 1**

The generalized equilibrium equation for linear situation can be expressed as:

\[
\iiint \mathbf{B}^t \sigma \, dv = F
\]

(13)

**Step 2**

The whole domain is divided into finite elements, which are connected together by specific nodes. Then the displacement vector \( \mathbf{U} \) at point \((x, y)\) can be expressed as follows:

\[
\mathbf{U}(x, y) = \sum_{i=1}^{N} \mathbf{u}_i \mathbf{N}_i(x, y)
\]

(14)

**Step 3**

The relation between the applied force acting on the nodes and the nodal displacement can be expressed by using what is called element stiffness matrix as follows:

\[
K_{op} = \iiint \mathbf{B}^t D_{op} B \, dxdy
\]

(15)

For elastic-plastic the D-matrix which relates plastic deformation, the increments of stress and strain can be expressed as follows:

\[
[D_{op}] = [D] - \frac{\partial \mathbf{H}}{\partial \sigma}^T \frac{\partial \mathbf{H}}{\partial \sigma} [D]
\]

\[
A + \frac{\partial \mathbf{H}}{\partial \sigma}^T \frac{\partial \mathbf{H}}{\partial \sigma}
\]

The D-matrix for plain strain elastic is:

\[
D = \frac{E}{(1-2\nu)(1+\nu)}
\]

\[
\begin{bmatrix}
1-\nu & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & 1-2\nu
\end{bmatrix}
\]

\[
\frac{1}{2}
\]
Effect of Residual Stresses on Elastic-Plastic Fracture Mechanics of Ductile Cast Iron Pipe under Pressure Loading

where,

\[
\frac{\partial H}{\partial \sigma} = \frac{\partial H}{\partial \sigma_m} + \frac{\partial H}{\partial \sigma_n} + \frac{\partial H}{\partial \sigma} + \frac{\partial H}{\partial \phi} + \frac{\partial H}{\partial \phi}.
\]

And

\[
\sigma_m = \frac{\sigma + \sigma_n}{2}, \quad \sigma = \left[ \frac{1}{2} \left( S_1^2 + S_2^2 + S_3^2 \right) + \tau^2 \right]^{0.5}, \quad \phi = \frac{1}{2} \sin^{-1}\left[ -\frac{3\sqrt{3} J_3}{2 \sigma^3} \right]
\]

\[
A = \frac{1}{\lambda} \int \left( \frac{\partial H}{\partial R} \right), \quad H = 2\sigma \cos \phi - \sigma_m, \quad S_1 = \sigma_m - \sigma_m, \quad S_3 = \sigma_3 - \sigma_m, \quad J_3 = S_3 S_3 - S_1 \tau^2
\]

Step 4

The nodal stiffness and nodal loads for each of the elements sharing the same nodes are added to each other to obtain the net stiffness and the net load at the specific nodes, so the global matrix can be expressed as:

\[
K = \sum_{i=1}^{n_e} K_{(x=x,y)} \quad (16)
\]

\[
P = \sum_{i=1}^{n_e} P_{(x=x,y)} \quad (17)
\]

Step 5

In this case, a pressure force is assumed to act on n-nodes of surface element, then the general expression for pressure force at point \((\zeta, \eta)\) is:

\[
F_{\zeta} = -P_{\zeta} \int N_i J_i d\zeta \quad (18)
\]

\[
F_{\eta} = +P_{\eta} \int N_i J_i d\zeta \quad (19)
\]

Step 6

The overall system of equation of the domain can be written as:

\[
[K] [U] = [F] \quad (20)
\]
In order to solve the above system of equations, the following boundary conditions are applied:

1- At loaded nodes the displacement is unknown and the applied force is known.
2- At supported nodes the load is unknown and the displacement is known.

Step 7
After applying the two boundary conditions, the system of equations is reduced and can be partitioned as:

$$K_u U_u + K_p U_p = F_u$$  \hspace{1cm} (21)  \\
$$K_m U_u + K_m U_p = F_p$$  \hspace{1cm} (22)

To solve Equations 21 and 22, the Gauss-elimination solver can be used.