Mixed – Symmetry States in Hg Isotopes using Interacting Boson Model (IBM)

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Abstract

Low-lying states of Hg isotopes are investigated in terms of the Interacting Boson Model (IBM). Mixed-symmetry states with neutron and proton bosons are characterized as 1−, 2− which are strongly excited by M1 transitions from the ground state. The states 1− and 2− have been shifted up out of the low-energy spectrum, as desired. However, the calculated 3− states, which were already too high (Fig. 1), have also been pushed up. These 3− states, along with the 2− and 4− members of the quasigamma of 198−202Hg [11,12] have found ξ− = 0.0 and ξ− of the order of −0.1 or −0.2 MeV. This is in reasonable agreement with the experimental results for the 3− states.

Introduction

The interacting boson model (IBM) of Arima and Iachello [1, 2] had a clear success in describing the low-lying collective states of several medium and heavy mass nuclei such as Hg isotopes (Z = 80, 114 < N < 122). In the collective Hamiltonian, each pair of proton and neutron can be coupled to L = 0 (S–boson) or L = 2 (d–boson).

We consider an even–even nucleus with Np (proton) and Nn (neutron) pairs as an interacting system. The number of pairs is counted from the nearest closed shell. For example, in 200Hg there are 1 proton boson hole (Np = 1) and 3 neutron bosons (Nn = 3). To obtain the energy spectra of the Hg isotopes, an appropriate set of parameters in IBM–Hamiltonian is used. One of the interesting features of this Hamiltonian is the prediction of a new set of collective states of which the quadrupole...
degree of freedom of neutrons and protons are excited in a different manner [3, 4]. The calculations of IBM are compared with the available experimental data [5].

**The Interacting Boson Model**

We consider a system of $N_p$ proton bosons and $N_v$ neutron bosons outside a closed shell and assume that these bosons do not interact with the bosons inside the closed shell (the core bosons). The contribution of the core to the Hamiltonian is then merely a constant term. So we write the Hamiltonian in terms of the valence bosons only:

$$H = H_{\Pi} + H_v + V_{\pi v},$$  \hspace{1cm} (1)

where $(H_\pi, H_v)$ represent the single-boson energy and boson-boson interactions for the proton (neutron) bosons, and $V_{\pi v}$ is the interaction between the proton and neutron bosons.

These energies and their interactions can, in principle, be derived from microscopic theory [2, 6]; but in this work the problem has been treated phenomenologically.

We also incorporate the essential features of the underlying microscopic picture in fermion space. These features have a strong pairing force between identical particles and a strong quadrupole interaction between nonidentical particles [1, 2]. In the phenomenological Hamiltonian of the form

$$H = \varepsilon (n_{d\pi} + n_{d\nu}) + K Q_{\pi} Q_{\nu} + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu}$$  \hspace{1cm} (2)

the first term takes into account the strong pairing assume to be equal for protons and neutrons $(\varepsilon_{\pi} = \varepsilon_{\nu} = \varepsilon)$, while $n_{d\pi(v)}$ is the number operator for proton (neutron) boson, the second term stresses the quadrupole character of the interaction between protons and neutrons:

$$Q_p = (s^+ d^- + d^+ s^-)^{(2)}_p + \chi_p (d^+ d^-)^{(2)}_p, \hspace{0.5cm} P = \pi \text{ or } \nu$$  \hspace{1cm} (3)

and

$$V_{pp} = \sum_{I=0,2,4} C_{I,p} \left[ (d^+ d^-)^{(I)}_p (d^+ d^-)^{(I)}_p \right] [0^1]$$  \hspace{1cm} (4)

where $V_{\pi\pi}$ and $V_{\nu\nu}$ represent protons (neutrons) boson interactions, respectively, which are extracted from microscopic calculations, and $M_{\pi\nu}$ is a Majorana force of the form.

-24-
Mixed-Symmetry States in Hg Isotopes using Interacting Boson Model (IBM)

\[ M_{\pi\nu} = \sum_{l=1,3} \xi_l (d_{\nu}^l d_{\pi}^m)^{1/4} \cdot (d_{\nu}^l d_{\pi}^m)^{1/4} + \xi_2 (s_{\nu} d_{\pi}^l - s_{\pi} d_{\mu}^l)^{1/4} \cdot (s_{\nu} d_{\pi}^l - s_{\pi} d_{\mu}^l)^{1/4}. \]

However, this calculation points out that, in the case of the Hg isotopes, the effects of \( V_{\nu\gamma} \) are completely negligible, whereas \( V_{\pi\nu} \) affects only minor details of the spectrum. Finally, the Majorana force \( M_{\pi\nu} \) shifts the states of mixed proton-neutron symmetry with respect to the totally symmetric ones. Since very little experimental information is known about such states with mixed symmetry, we have not attempted to fit the parameters appearing in Eq. (4).

Results and Discussion

1 Model's Parameters

In principle, all parameters can be varied independently in fitting the energy spectrum of one nucleus. However, in order to reduce the number of free parameters in agreement with microscopic calculations of Ref. 7, only \( \nu \) and \( K \) vary as a function of both \( N_{\pi} \) and \( N_{\nu} \): i.e., \( \nu = (N_{\pi}, N_{\nu}) \) and \( K = K (N_{\pi} - N_{\nu}) \) are allowed. The other parameters depend only on \( N_{\pi} \) or \( N_{\nu} \): i.e.,

\[ \chi_{\pi} = \chi_{\pi} (N_{\pi}); \chi_{\nu} = \chi_{\nu} (N_{\nu}); C_{L,\pi} = C_{L,\pi} (N_{\pi}); \text{and} C_{L,\nu} = C_{L,\nu} (N_{\nu}). \]

Thus, in the isotopes chain, \( \chi_{\pi} \) is kept constant, whereas for two isotonic Hg nuclei, \( \chi_{\nu}, C_{L,\pi}, \text{and} C_{L,\nu} \) are kept constant, as shown in Table (1).

2 Energy Spectra

New experimental energies of the O' states in some of the heavy Hg isotopes [8] are different from the previously reported values [5] as shown in Fig 1. Previous data [9] show a decrease in the energy of the O' states as the neutron number decreases. The new data which do not show this trend can be reproduced with a constant value of \( C_{0,\nu} = 0.4 \text{MeV} \). The O' states of Vergnes et al. [8] and the theoretical values are also shown in Fig 1.

It is found that a collective 1' state in \(^{156}\text{Gd}\) [10] indirectly affects the mercury calculations because it suggests that the strengths of the Majorana terms used in most of the IBM-2 calculations to date have been too small. This 1' state was found at 3.1 MeV in an inelastic electron scattering experiment \((e, e')\), which selectively excites collective states.

In the interacting boson model (IBM), 1' states are clearly not totally symmetric, and cannot be obtained in IBM-1. Thus, they are quite sensitive to the strength of the Majorana force. The M1 state in \(^{156}\text{Gd}\) can be reproduced by IBM-2, with the
Majorana strengths $\xi_2 = 0.2\, MeV$ and $\xi_3 = 0.4\, MeV$. The values of the parameters $\xi_{1,3}$ used by these workers are also much smaller than of Bohle et al.\cite{10}.

Although some of the heavy mercury isotopes have low-Lying $1^+$ states, which have no evidence of collectivity, the calculated $1^+$ states should be higher in energy than those of Ref.\cite{9} - indicating that one or more of the parameters $\xi_{1,2,3}$ need to be increased in absolute value.

Another probable indication for the need of increasing the Majorana force in the isotopes calculation is the presence of the mixed - symmetry $2^-$ states (Fig 1). No experimental evidence for such states, except near the closed shell, where the $2^-$ state could be non collective, has been found.

The states $1^-$ and $2^-$ have been shifted up out of the low-energy spectrum, as desired. However, the calculated $3^-$ states, which were already too high (Fig 1), have also been pushed up. These $3^-$ states, along with the $2^-$ and $4^-$ members of the quasigamma band, are shown on the right-hand side of Fig. 1. Recent microscopic calculations of $^{198-202}$Hg\cite{11,12} have found $\xi_2 = 0.0$ and $\xi_3$ of the order of -0.1 or -0.2 MeV. This is in reasonable agreement with the experimental results for the $3^-$ states. The $2^-$ states of Durce et al.\cite{11} are around 1.5 MeV.

Table 1. IBM-2 Parameters for Hg Isotopes in (MeV) Units

<table>
<thead>
<tr>
<th>NUCLEI</th>
<th>$\varepsilon$</th>
<th>K</th>
<th>$C_0^\nu$</th>
<th>$C_2^\nu$</th>
<th>$C_4^\nu$</th>
<th>$X_\pi$</th>
<th>$X_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hg-194</td>
<td>0.620</td>
<td>-0.19</td>
<td>0.402</td>
<td>0.20</td>
<td>0.151</td>
<td>0.65</td>
<td>-0.4</td>
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<td>-0.20</td>
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<td></td>
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<tr>
<td>Hg-198</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Hg-200</td>
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<td>-0.22</td>
<td></td>
<td></td>
<td></td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Hg-202</td>
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<td>-0.23</td>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>1.15</td>
</tr>
</tbody>
</table>

$\xi_1 = \xi_3 = -0.4$

$\xi_2 = 0.2\, MeV$
Fig. 1: Comparison between Experimental and Calculated Energy – Spectra for Hg Isotopes
أوضاع التماثل المختلة لنظام الرنين باستخدام نموذج البوزونات المتفاعلة (IBM)

محمد قاسم الفخار

ملخص

استخدم برنامج الحاسب الآلي المسمي نموذج البوزونات المتفاعلة (IBM) ونلدلقي دراسة حالات مستويات الطاقة الدنيا لنظام الرنين، حيث أن الهدف على المستوى البعيد هو فهم معطيات النموذج بدلالة النظرية المجهرية مثل النموذج النووي الافتراضي.

وقد تبين أن أوضاع التماثل المختلة لنموذج البوزونات والبيروتونات للحالات M من الحالة الأرضية المتهجَّة بشدة بانتقال +1.2 من الطريقة المختلَّة.

إذاً، أن المستوى A1,2 ينحرف إلى الأعلى خارج طيف الطاقة المنخفض، وكذلك يوجد المستوى المحسوب 3- والذي تعتبر أصلاً عالية قد ارتفعت هي بدونها أيضاً. 

وقد من الحسابات المجهرية التي تم إجرائها من قبل بعض الباحثين على نظام الرنين Hg²⁰⁶ وجد أن 0.0 ≤ H ≤ 0.2، وكذلك يوجد أن H يتراوح بين 0.1-0.2. م. أ. ف. أن هذا يدل على وجود اتفاق مع الفن القياسي لمستويات 3.

References

5. Lederer and Shirley, Table of Isotopes 7th edition.
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