Relaxation Rates of Three-Dimensional Electron Gas in a Magnetic Field

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Abstract

The relaxation in Fermi liquids is a kinetic phenomenon which may be attributed to several channels among these: (a) relaxation due to electron-electron (e-e) scattering; (b) the relaxation due to the emission of phonons; (c) elastic scattering due to interface roughness. The relaxation rates of three dimensional (3D) electron gas as a function of magnetic field has been derived from time-resolved optical spectra. A theoretical model of the electron scattering from the interface roughness has been developed. A single relaxation time \( \tau \) corresponding to the decay of particles near the Fermi surface turns out to be dominant. A good agreement between experiment data and model calculations has been achieved.

I. Introduction

The relaxation of electron gas is a kinetic phenomenon which may occur through one or more of the following channels: (a) relaxation due to the emission of phonons; (b) relaxation due to the electron-electron (e-e) scattering (in this process the excess energy is transferred to the 3D electrons); (c) intersubband elastic scattering due to impurities followed by intersubband energy relaxation; (d) elastic scattering due to the interface roughness also followed by the intersubband relaxation.

Mechanisms (a) and (b) were considered in detail, in [1]. The experimental results indicate that the electron-phonon scattering may lead to a considerably slower relaxation: \( \tau_{e-ph}^{-1} \sim 5 \times 10^6/s \). The e-e scattering is most efficient if the Fermi level \( E_F \) is

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near \((0.3-0.35)\Delta_{10}\), where \(\Delta_{10}\) is the intersubband splitting. The sufficient condition for having e-e scattering is found to be \(\Delta_{10} - E_f << E_f\) \([2]\). The experimental relaxation rate for this process is found to be \((\tau_{ee}^{-1} = 5 \times 10^9/s)\) \([3]\).

The intersubband scattering due to impurities was discussed, for example, in \([4]\). The formalism of two dimensional (2D) electron scattering has been developed fairly well. Only those impurities which are placed at a distance \(L < (1/k_F)\) from the 2D channel make a considerable contribution to the electron scattering rate \([5, 6]\).

The relaxation of 2D electrons due to the elastic scattering from the interface roughness was considered in the absence of magnetic fields \([7]\). We have extended this theory to the case of intersubband scattering of 3D electrons in a parallel magnetic field.

The intersubband relaxation of 3D electrons is very important for many processes experimentally observed in a magnetic field. For example, the magneto-oscillations of the luminescence intensity due to electrons in the excited subband \([8]\) and oscillations of the transition time of 3D electron from the excited subband \([9]\) are due to intersubband relaxation. A small magnetic field affects the shape of the quantum well and electron wave function \([10]\), but it does not generate discrete levels in the electron spectrum, which allows us to obtain a simpler solution of the intersubband relaxation problem. Consequently, a realistic model of interface roughness scattering is essential for the interpretation of transport properties of electron gas at low-temperatures.

This paper is devoted for investigating the intersubband relaxation rate of 3D electron gas as a function of parallel magnetic field. Our attention will be focused on the transition of 3D electrons due to the interface roughness.

The rest of the paper is organized as follows: in Section 2 we introduce the theoretical model. Results are discussed in Section 3. The last Section is devoted for discussion and conclusion.

II. Theoretical Model

In this model we reexamine interface roughness scattering in quantum wells (QWs). In particular, we show how scattering matrix elements are altered when the finite depth of the QW is taken into account.

The basic assumption of the model is that the interface is a three dimensional potential well with slowly varying position \(z = \Delta(r)\). Such fluctuations of the barrier position can be caused either by the interface roughness or by the impurities of small concentration present in the spacer.

Let the energy intersubband splitting \(\Delta_{10}\) be large and changes in the wave function shape be negligible. In this case, the corresponding scattering matrix element of the intersubband transition in a small parallel magnetic field takes the form \([2]\):

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\[
M_{nkn'k'} = \int dz \int d\bar{r} e^{i\bar{k}_n \cdot \bar{r}} \left\{ \psi_n^*\left[z - \Delta(\bar{r})\right] \left[H_0 + \Delta V(\bar{r}, z) + \frac{m \omega_c^2}{2} (z - \Delta(\bar{r}))^2\right] \psi_{n'} \right\} \left[e^{i\bar{k}'_n \cdot \bar{r}}\right] \quad (1)
\]

In equation (1) \(H_0\) is the Hamiltonian in the case of zero roughness and magnetic field, \(\Delta V(\bar{r}, z)\) is the change in the potential energy due to roughness and \(\omega_c\) is the cyclotron frequency. In case of transition between states with equal energies (elastic scattering) and when the interface is a sharp infinite well, we obtain the matrix element in the linear \(\Delta(\bar{r})\) approximation in the form

\[
M_{nkn'k'} = \frac{\hbar^2}{2m} \Delta_{\bar{k}-\bar{k}'} \frac{d\psi_{n}^*}{dz} \frac{d\psi_n}{dz} \bigg|_{z=0} + j\int dz j d\bar{r} e^{i\bar{k}_n \cdot \bar{r}} \left[ -i(\bar{k} - \bar{k}') \right]^* \psi_n(z) \left[ \Delta V(\bar{r}, z) + m \omega_c^2 z \Delta(\bar{r}) \right] \psi_{n'}(z) \quad (2)
\]

where

\[
\Delta(\bar{r}) = \sum_q \Delta_q e^{i\bar{q} \cdot \bar{r}}
\]

In our specific calculation, we assume that the quantum well is cylindrical. This means that without a magnetic field the wave function is of Bessel's function type.

Next, we discuss the roughness auto-correlation function \(S(\bar{r})\) of the fluctuations in the QW width. The auto-correlation function is defined as [11]:

\[
S(\bar{r}) = \left\langle \Delta(\bar{r}) \Delta(\bar{r}') \right\rangle = \frac{1}{V} \int \Delta(\bar{r}) \Delta(\bar{r}') d^3 \bar{r}' = \Delta^2 e^{-\frac{\left(\bar{r} - \bar{r}'\right)^2}{\lambda^2}} \quad (3)
\]

where \(V\) is the normalization volume, \(\lambda\) is the correlation length (is the order of the characteristic dimension of irregularity in the interface plane) and \(\Delta\) is the average fluctuation height or the average shift of the interface. Such a form of correlation function is applicable when \(\Delta\) is small in comparison with the characteristic dimensions of the quantum well [12]. Taking the Fourier transform of the auto-correlation function in equation (3), we obtain

\[
S(\bar{q}) = \left\langle |\Delta_q|^2 \right\rangle = \pi \Delta^2 \lambda^2 e^{\frac{\pi^2 \lambda^2}{4}} \quad (4)
\]
The inverse correlation length $\lambda^{-1}$ corresponds to an effective cut-off wave vector for the momentum exchange $q$.

The relaxation time rate of an electron is given by the following expression [13]:

$$\frac{1}{\tau} = \frac{1}{2\pi} \int d^3k |M_{\alpha\beta}k|^2 (1 - \cos \varphi) \delta [E(k) - E(k')]$$ \hspace{1cm} \ldots(5)

where $\varphi$ denotes the angle between the initial and final wavelength $k$ and $k'$. If cylindrically symmetric subbands are assumed, equation (5) reduces to

$$\frac{1}{\tau} = \frac{\rho(k)}{\hbar k^2} \int d^3q |M_{\alpha\beta}k|^2 \frac{q^2}{\sqrt{4k^2 - q^2}}$$ \hspace{1cm} \ldots(6)

where $\rho(k)$ is the 3D density of states.

The relaxation rate of electron gas is obtained by substituting equation (2) into equation (6). After having some simple algebra, we obtain an equation for the relaxation time rate during which an electron can abandon the excited subband owing to the scattering from roughness:

$$\frac{1}{\tau} = \frac{2\pi m}{\hbar^2} \left[ \frac{4\pi e^2 n_1}{\varepsilon} + 2m \omega_0 \left( \frac{\hbar^2 e}{8\pi e^2 n_1 m} \right)^{1/3} \alpha \right] \Delta^2 \lambda^2 e^{-\Delta_1 \lambda^2 / 2k^2} \left( \frac{\alpha^2 \lambda^2}{\Delta_0 \lambda^2} \right) \left( \frac{\lambda^2}{\Delta_0 \lambda^2} \right)$$ \hspace{1cm} \ldots(7)

where $\Delta_{10}$ is the energy splitting without a magnetic field, $\varepsilon$ is the dielectric constant and $\alpha$ and $\beta$ are the dimensionless constants having values of $\alpha=0.85, \beta=1/2(\hbar^2/\Delta_1 \lambda^2)(<\varepsilon^2> - <\varepsilon>^2)=0.056$ [14].

For a weak magnetic field $B=2(n,0,0)$, the electrons distribution function can be approximated by a shifted 3D Fermi sphere. One obtains the mobility of the electron gas

$$\mu = e \int \frac{dE \rho(E) v^2}{4nk_BT \cosh^2 \left[ \frac{E - \mu}{2k_BT} \right]}$$ \hspace{1cm} \ldots(8)

where $\mu$ is the Fermi energy, $n$ is the electron density and $v$ is the electron velocity.

**III. Results**

The experimental data of the Fermi gas relaxation rates were obtained from time-resolved luminescence spectra due to recombination of electrons from the
Excited subband with holes generated by light and bound at the delta layer acceptors. A sample containing a single GaAs-AlGaAs heterostructure and a delta (δ) layer of Be acceptor atoms with a density of $2 \times 10^{10}$ cm$^{-2}$ in the GaAs buffer layer at 30 nm from the junction was used in investigating the 2D Fermi gas relaxation rates [15]. The sample was placed in a liquid helium with a superconducting solenoid, and the experiments were performed in a magnetic field aligned with the junction interface at a temperature of 4.2 K. The sample was excited by a picosecond laser system generating pulses with a width of 1 ps at a wavelength of 600 nm, laser light being conducted into the sample by means of optical fibre. Luminescence was shot into the same fibre and fed to the input slit of a grating monochromator with a linear dispersion of 1.3 nm. A time correlated photon counting system is being used to record both recombination spectra at a fixed delay and luminescence decay at a fixed wavelength. The relaxation rates derived from the experimental data is plotted against the parallel magnetic field in Figure 1 (dashed line).

The theory was applied to electrons in strained Si QWs of thickness 6 nm and AlGaAs spacer of thickness ~3.5 nm. We have estimated the relaxation rate (7) for the parameters $\Delta_{12}=15$ meV, $m=0.067 m_e$ and $\varepsilon=13$ of our structure and at electron concentrations namely, $n_e=3.5 \times 10^{11}$ cm$^{-3}$ and $n_e=3.8 \times 10^{11}$ cm$^{-3}$ given above. The results are shown in Figure 1 for roughness parameters $\Delta=0.3$ nm, $\lambda=4$ nm. Due to the lack of detailed experimental information, these parameters have usually entered transport calculations as fitting parameters. For the parameters chosen in this work, interface roughness scattering is found to be a limited scattering mechanism in electron QWs thinner than 2 nm. For thicker QWs, contribution of other scattering mechanisms, such as electron-phonon scattering mechanism to the relaxation rate can not be neglected.

The estimated relaxation rate of this model indicates that the electron-phonon scattering may lead to a considerable slower relaxation time: $\tau_{e-ph}=2.4 \times 10^7$ s. The electron-electron scattering process relaxation rate is found to be $\tau_{e-e}=1.4 \times 10^8$ s. This indicates that the e-e process may be dominant over the e-ph scattering process in agreement with experimental results [15]. Finally, the estimated relaxation rates due to impurities yield $\tau_{imp}=2.4 \times 10^7$ s which disagrees with the experimental data $\tau_{imp}=0.4 \times 10^8$ s [16]. This maybe attributed to the limited number of scattering centers. In other words, the structure we have considered does not have enough scattering centers in 3D well because the impurities are separated from the interface by an AlGaAs spacer with thickness of ~3.5 nm. This restricts the number of impurities which make a considerable contribution to the electron scattering and the corresponding relaxation rates.

For these parameters of the average roughness, we have a transport mobility of the order $6.8 \times 10^5$ cm$^2$/V/s, according to the calculation for the intersubband scattering due to roughness. It is not surprising that the calculated transport mobility is fairly high because in experiments the transport mobility is limited by the angle of scattering from the impurities behind the interface. For the density of $n_e=2 \times 10^{10}$ cm$^{-3}$ and spacer widths of 20 nm, 10 nm, 5 nm the calculated mobility for Si range from
4.5 \times 10^4 \text{ cm}^2/\text{Vs} \text{ to } 2.5 \times 10^3 \text{ cm}^2/\text{Vs}.

In general, the calculated intersubband scattering due to roughness of the interface are found to be in good agreement with experimental data for small parameter values and deep QWs.

IV. Conclusion And Discussion

In this paper we have developed a simple model for investigating the intersubband electron scattering from the interface roughness. The intersubband relaxation rate of 3D electrons as a function of a parallel magnetic field is derived. We have demonstrated that the main channel of the intersubband relaxation in the studied structure is electron transitions from the excited subband due to the elastic scattering from the interface roughness.

Interface roughness scattering can exceed all other scattering processes in narrow quantum wells. The theory is good enough to calculate low-temperature mobility.

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معدلات الترخاخ للفاز الإلكتروني في ثلاثة أبعاد بدلالة المجال المغناطيسي
محمد أبوصرة

ملخص

يعرف الترخاخ في مائع فيرمي على أنه ظاهرة انتقالية حركية تميز إلى عدة سبل أو فنوات. منها: (أ) الترخاخ بسبب تشتت الإلكترون مع الإلكترونات (ب) الترخاخ نتيجة الابتعاد الفيزيائي (ت) الترخاخ بسبب التصادم المرئي والذي ينتمي إلى خصوبة السطح الفاصل. لقد تم استنتاج معدلات الترخاخ للفاز الإلكتروني في ثلاثة أبعاد بدلالة المجال المغناطيسي من التحليل الزمني للطيف الفيزيائي وتم تطوير نظرية لدراسة التشتت الإلكتروني من خصوبة السطح الفاصل. وأثبت النتائج سرعة زمن ترخاخ واحد له علاقة بالتفاعلات التفوقية في عدد الجسيمات قرب سطوح فيرمي. وبدأت الدراسة أن هناك تطابقا وتوافقا جيدا بين النتائج العملية والنظرية.

References

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(1998), 705.


Figure 1. The relaxation rates versus parallel magnetic field.

--- Experimental results.

--- Fits described by equation (7).

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