Parameter Estimation of Hierarchical Linear Models by Using Generalized Maximum Entropy

By

Luay Mahmoud Mohaidat
B.Sc. Statistics Sciences, Yarmouk University, 2002

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Approved by:
Dr. Amjad D. Al-Nasser..........................Chairman.
Associate Professor of Statistics, Yarmouk University.

Dr. Omar M. Eidous..........................Co-supervisor.
Associate Professor of Statistics, Yarmouk University.

Dr. Maref Al-zoubi..........................Member.
Associate Professor of Mathematics, Yarmouk University.

Dr. Mohammed Y. Al-Rawwash..........................Member.
Associate Professor of Statistics, Yarmouk University.

Dr. Mohammed H. Baker Al-Haj Ebrahim..............Member.
Associate Professor of Statistics, Yarmouk University.

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ABSTRACT


In simple linear regression all effects are modeled to occur at a single level. However, when the data is nested or has clustered structure then advanced regression form is needed, in which lower-level units of analysis are nested within higher-level units of analysis; such multilevel models are well-known as Hierarchical Linear Models (HLM). HLM is a generalization of linear regression model, which can be used for a variety of purposes, including prediction, data reduction and causal inference from experiments and observation studies.

There are many estimation methods used to fit the HLM such as Empirical Bayes (EB) and Ordinary Least Squared (OLS) methods. However, using these estimation methods for the same data can produce rather different point and interval estimates, and the applied multilevel modeler may be left wondering what to report.

The main purpose of this study is to look at the Generalized Maximum Entropy (GME); as a new methodology for parameters estimation in HLM. Our investigations were focused in two level models; the only-intercept model and the random coefficients model. The theoretical results showed that the GME estimators are unbiased. Also, the numerical
results by using Monte Carlo simulation experiments provide some promising results about the superiority of the GME estimators over both EB and its modifications “Morris and Rao”; and over the OLS estimators when the HLM is considered.

The final episode of this study presents an application on education research based on the existing data from High School and Beyond. A two level HLM was developed to study why some schools have higher means (in math-achievement) than others and why in some schools the association between socioeconomic status (SES) and math-achievement is stronger than others. The empirical results based on GME and OLS methods indicated that the mean of SES is positively related to school mean of math-achievement. In addition, private schools have higher mean achievement than public schools, controlling for the effect of SES mean.

Key Words: Generalized Maximum Entropy, Hierarchical Linear Model, Ordinary Least Squares, Empirical Bayes.