BAYESIAN AND NON-BAYESIAN ESTIMATION USING DATA IN A FREQUENCY TABLE WITH APPLICATION TO EXPONENTIAL DISTRIBUTION

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B.Sc. (Statistics)
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1995

Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science (Statistics) at Yarmouk University

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ABSTRACT

In this thesis we consider the problem of estimating the parameter $\theta$, when a random sample of size $N$ from $f(x|\theta)$ is summarized in a frequency table. In particular, we are interested in the estimation of the parameter $\theta$ of the exponential distribution. Using classical approach we obtain five estimators of $\theta$. These estimators are then compared with three well-known estimators of $\theta$ (namely, the MLE, the grouped sample mean, the mean of the sample simulated uniformly from the intervals).

For the exponential distribution, it is shown that the Fisher information for the data in a frequency table is always less than the Fisher information for the raw data. The loss of information in using the frequency table instead of the raw data is investigated.

A formula that relates the posterior density function given the data in a frequency table to that given the raw data is used to relate the two corresponding Bayes estimators, compare with respect to their posterior variances and Bayes risks. It is shown that the Bayes risk using grouped data is always larger than the Bayes risk using actual data provided that the prior distribution of $\theta$ is the same in
both cases. The same formula is also used to approximate Bayes estimators.

Using different types of priors for $\theta$ we obtain four Bayes estimators of $\theta$. These estimators are compared with other related estimators. For some of these estimators, the loss of information, measured by a suitable measure that is a function of the Bayes risk, is not too high when the number of intervals is sufficiently large. In the end of each chapter these estimators are calculated for an artificial data set given in a frequency table. Real data is used in the last chapter to calculate the value of each estimator.