On the power function of the LRT for restricted alternatives in the case of Multivariate Normal distribution

By
Amjad Deafalih Al-Naser
B. Sc. (Statistics), 1990
Yarmouk University

Thesis submitted in partial fulfillment of the requirements for the degree of Master of Science (Statistics) at Yarmouk University

Thesis Defense Committee:

Dr. Hussain Al-Rawwash

Prof. Mohammad S. Ahmed

Dr. Waleed Abu-Dayyeh

Dr. Mohammad Fraiwan

Chairman:
Member:
Member:
Member:
ABSTRACT

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Let the random vector \( X \) be distributed as a multivariate normal with mean vector \( \Omega = (\theta_1, \theta_2, \ldots, \theta_p) \) and identity covariance matrix. We will consider testing a hypothesis of the form

\[
H_0 : \theta = \Omega \ vs \ H_1 : \theta \in V_\gamma \setminus \{\Omega\}
\]

where

\[
V_\gamma = \{\Omega : \theta_i \geq 0 \ \forall i = 1, 2, \ldots, J\} \cup \{\Omega : \theta_i \leq 0 \ \forall i = 1, 2, \ldots, J\}, J \leq p
\]

The thesis is divided into two parts. In the first part, we derive the LRT and its power function for the case \( p = 3 \) and \( J = 1, 2, 3 \). Then we generalize the result for any \( J \leq p, J > 3 \). Also, we derive some properties of the power function.

The second part is devoted to the derivation of a family of Bayes combination tests using two types of prior densities for the case \( p = J = 3 \). Then we compare numerically the power function of the LRT when \( p = 3, J = 1, 2, 3 \) and the Bayes combination tests at significance level \( \alpha = 0.05 \).

From the numerical computations it appears that, the more restrictions imposed on the alternative space, the more powerful the LRT becomes. Furthermore, it turns out that Bayes combination tests have decreasing power in \( \alpha \in [0, \frac{\pi}{4}] \) and increasing power in \( \alpha \in [\frac{\pi}{4}, \frac{\pi}{2}] \) for fixed \( \lambda \) and \( \gamma \) when using type I prior, and vice versa when using type II prior.