Some Integrals Involving Bessel Functions

Ahmad Rasheed Abd Al-Rahman Al-Momani

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Yarmouk University

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University.

Thesis Defence Committee:

Dr. Ali Al-Jarrah  
Dr. Othman Malhas  
Prof. Nabil Laha
Dr. Walid Abu-Dayyeh  

Chairman:  
Member:  
Member:  
Member:  

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Abstract

Bessel Functions are among the most important special functions, having diverse applications to physics, engineering and mathematical analysis. In particular, the integral representations for the product of Bessel functions are useful for evaluating definite integrals that contain products of two Bessel functions under the integral sign. In addition, they may be considered as a natural generalization of well-known trigonometric identities.

In the first part of this thesis we prove some known integral representations for the product of two Bessel functions of the same argument and same order, one of which is Nicholson's formula, which expresses the sum $J_0^2(x) + Y_0^2(x)$ as an integral over a hyperbolic Bessel function.

Sonine-Gegenbauer type integrals are important in some physical applications. Some of these integrals have been evaluated in terms of hypergeometric functions of two and three variables. In the second part of this thesis, we evaluate some special cases of these integrals in terms of familiar functions.

In the third part of this thesis, we investigate a special class of Bessel integrals, namely we study the class of integrals $\int_0^\infty t^{v+1}J_v(at)\frac{F(t)}{t^{1+b}r^m}dt$, where $F(t)$ is a function that satisfies certain conditions. In fact, we state and prove
some theorems regarding this class of integrals.

Key Words: Bessel Functions, Infinite Integrals, Integral Representations.