Bayesian Estimation of the Lorenz Curve and Gini index for the Pareto model

By

Mohammed Ahmed Obeidat

B. Sc. (Statistics)

Yarmouk University

Irbid-Jordan

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Thesis Defense Committee

Dr. Ayman S. Baklizi    Chairman

Dr. Walid A. Abu Dayyeh    Member

Dr. Mohamad F. Al-Saleh    Member

Dr. Mahmoud M. Al-Smadi    Member

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ABSTRACT

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size $n$ from the Pareto distribution with probability density function

$$f(x|\alpha, \beta) = \beta \alpha^\beta x^{-(\beta+1)}, x \geq \alpha > 0, \beta > 1$$

The (generalized) Bayes estimators of the Lorenz curve and Gini index in case of the Pareto Model, using informative as well as non-informative prior, are derived when the squared error loss function, is used.

The Bayes estimators in this case can not be obtained in closed form, so we consider some methods of approximations of these estimators, namely Lindley, Tierney-Kadane (T-K) and Gibbs sampler procedures.

These approximations are studied and compared by simulation, the comparison are based on the square distance between exact Bayes estimator and its approximation. The distance is defined as $E_{X|\theta}[(\hat{\theta} - \theta)^2]$, where $\theta$ is the Bayes estimator and $\hat{\theta}$ is its approximation.
In case of non-informative prior, Gibbs sampler approximation has the smallest distance from the (generalized) Bayes estimator, followed by Lindley. In case of informative prior, and for the Lorenz curve, in general, the distance increases as \( p \) increases, and it decreases as \( c \) increases. Lindley has the smallest distance for most of the cases considered, but T-K and Gibbs sampler are approximately the same.