Finite Element Method for the Prediction of Pump Suction Performance at Developed Cavitation

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Abstract

In the present paper an attempt was made to apply the finite element method for the solution of the potential flow Laplace equation pertinent to flow conditions prevailing in a confined cascade channel which resembles the flow at centrifugal pump impeller entrance. By estimation of the stream function at different locations inside the channel, it was possible to obtain theoretically the values of the cavitation number at areas of minimum pressures.

This procedure allowed to evaluate critical values of the NPSH and suction specific speed of the pump at the beginning and at advanced stage of cavitation. Comparison of these results to values obtained by other authors, both theoretical and experimental results showed good agreement within the test range attempted.

Introduction

Suction performance of centrifugal pumps is a complex function of many variables such as liquid physical properties, flow velocity and orientation, NPSH and inlet geometry. As long as cavitation build up at impeller entrance is avoided, the required NPSH for satisfactory operation at normal flows may be predicted for given design with some accuracy. However when a fixed cavity of some size starts to build up at blade entrance, then it becomes rather difficult to predict suction performance such as NPSH and NSS and the problem becomes rather involved, requiring testing of the pump at near real life condition which is an expensive exercise.

Several researcher (Minamis, S. etal(1960), Yedidiah, S(1972) and Pearsall, I.S (1973)) attempted to estimate the values of the blade cavitation number at developed cavitation (3-5% head drop) for already available data obtained from many pumps.
and pump tests. The values however seem to be scattered over a wide range which makes it not suitable for the prediction of accurate NPSH at the design stage.

Better correlation were obtained from tests on hydrofoil cascades (Numanchi, F et al (1952) and Knapp, T et al (1970)) which resemble the flow in pump channels and at the same time avoid secondary effect like prerotation and film separation. Reports from NACA test section (Minami, S et al (1960)) showed that incident angle is the most important single variable which determine the cavitation number at inception point.

Theoretical prediction of cavitation number at developed cavitation are scarce. An attempt to predict blade cavitation number using a linearized potential flow theory for two dimensional flow between two flat cascades gave some promising results although non conclusive (Pearsall, S (1972)). A more recent work (Kueny, J. L., et al (1992)) which applies numerical iterative technique to solve

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the Eulerian equation by a 3D model on developed attached cavities in a centrifugal impeller channel gave more consistent results with some promising applications.

Finite element numerical solutions to complex fluid mechanics problems have been elaborated during the last 2 decades with some good results (Chung, T.J. (1978) and Abdul Wahab, M. (1995)). For the incompressible liquid, quadrilateral two-dimensional isoparametric element was found suitable for the solution of the Laplace and Stock equation of the flow field.

2- Suction parameter.

Suction capability of a centrifugal pump is conveniently expressed by the net positive suction head (or energy) NPSE, and the suction specific speed NSS. In terms of the external characteristics of the pump, they may appear in the following form.

\[ \text{NPSE} = \frac{1}{\phi} \left( P_T - P_v \right) \]  
(1)

\[ \text{NSS} = \frac{\omega \sqrt{Q}}{\text{NPSE}^{3/4}} \]  
(2)

Where \( P_T \) is the total pressure at the suction flange and \( P_v \) the vapour pressure at the prevailing temperature. Obviously the NPSE and NSS are related in such a way as to give higher suction specific speed for the lower NPSE with improved suction capability. Hence from its definition the suction specific speed expresses the ability of the pump of transferring liquid at the permissible NPSH required.

Eq. (1) describes the available NPSE at the suction flange as depicted by the operating system and for satisfactory operation it should not be less than that required by the pump as limited by design configuration which is normally given by the following equation:

\[ \text{NPSE} = A \frac{C_m^2}{2} + B \frac{U_1^2}{2} \]  
(3)
\( C_{m1} \) represents the flow velocity and \( U_1 \) the centripetal action or velocity at blade inlet. By application of the energy equation between the suction flange and blade inlet (Chalaby, A.A. (1997)), it is easy to show that \( B \) of Eq. (3) is equal to a blade cavitation number \( \sigma_b \), defined by:

\[
\sigma_b = \frac{p_1 - p_x}{\frac{1}{2} \varphi W_i^2} = B
\]

(4)

and that \( A \) is given by:

\[
A = 1 + K_C + \sigma_b
\]

(5)

where \( K_C \) is a contraction factor which depends on the smoothness of flow entry into the impeller. From experience, for advanced cavitation (Yedidiah, S. (1972) and Pearsall, I.S. (1973)) \( A \) was found to vary between 2 and 3 and \( B = \sigma_b \) in the range of 0.06 and 0.15, whereas theoretically \( \sigma_b \) may reach a value as high as 0.5.

Ignoring \( K_C \) for being small in most cases where the entry is streamline, the NPSE of Eq. (3) may be expressed in terms of the blade cavitation number, and other flow parameters in the following way (Chalaby, A.A. (1997))

\[
\text{NPSE} = \frac{1}{2} \left[ (1 + \sigma_b) \left( \frac{Q}{\pi} \left( \frac{\Omega}{(1 - \lambda)^2 D_{tl}} \frac{D_{tl} \omega}{2} \right)^2 + \sigma_b \left( \frac{D_{tl} \omega}{2} \right)^2 \right) \right]
\]

(6)

\( \lambda \) being the hip to tip diam. ratio (\( \lambda = (D_{in} \text{ } - \text{ } D_{lt}) \)) and is a strong function of the specific speed \( N_s \).
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By combination of known parameters at impeller inlet and making use of the flow coefficient \( \phi = C_{m_{i}}/U_{1} \), the NSS equation of Eq. (2) after the introduction of NPSE from Eq. 6 becomes:

\[
\text{NSS} = \frac{6\sqrt{1-\lambda}}{\phi \left[ \left(1 + \sigma_{b}\right) + \frac{\sigma_{b}}{\phi^2}\right]^{3/4} \sqrt{1 + \lambda}}
\]

Which is a useful equation describing the suction specific speed as a function of non-dimensional design parameters such as flow coefficient, cavitation number and diameter ratio. Optimization of NSS with respect to inlet diameter \( (D_{i}) \) would yield the optimum value for the flow coefficient:

\[
\phi_{\text{opt}} = \left[\frac{(1-\lambda^2)\sigma_{b}}{2(1+\sigma_{b})}\right]^{1/2}
\]

The optimum NSS may be found by inserting \( \phi_{\text{opt}} \) of Eq. (8) into Eq. (7):

\[
(NSS)_{\text{opt}} = 4.22\left(\frac{1-\lambda^2}{3-\lambda^2}\right)^{1/2} \left(\frac{1}{\sigma_{b}\sqrt{1+\sigma_{b}}}\right)^{1/2}
\]

Obviously \( (NSS)_{\text{opt}} \) of Eq. (9) cannot be realized, not only because of \( \phi_{\text{opt}} \) but also due to the presence of dynamic factors such as the contraction factor \( K_{C} \), the restriction factor imposed by the blade thickness and by the meridian flow velocity variation across the entry.
3- Theoretical model

From above reasoning it is obvious that the blade cavitation number $\sigma_b$ plays a dominant role in the determination of the NSS and NPSH of a pumps of known geometry. Therefore it would be highly desirable to find an easy way of predicting $\sigma_b$ with some accuracy, using available theories and practices. In this paper a theoretical model is chosen in which flow through a two-dimensional axial flat plate cascade of hydrofoils as a representation of the pump blades is used (Fig. 1). The cavity L is finite and less than the chord length c and is non-dimensionalized. The entrance length and angle are specified beforehand.

The problem is to solve the equation of continuity in two dimensions applying a numerical finite element method for the solution of the Laplace equation in terms of the stream function for a steady two dimensional potential flow, using Galerkin weighted residual method.

From potential flow theory, the velocities in the x and y directions are given by:

$$
\begin{align*}
  u &= \frac{1}{b} \frac{\partial \psi}{\partial y} \\
  v &= -\frac{1}{b} \frac{\partial \psi}{\partial x}
\end{align*}
$$

Where b is the depth of the channel. The continuity equation is defined by:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
$$

and for irrational flow,
\[
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \quad (12)
\]

and hence by substitution, the Laplace's equation is obtained:

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad (13)
\]

The domain is established as shown in (Fig. 2) with boundary conditions:

\[
\psi = \psi_0 \quad \text{on} S_1 \quad (14)
\]

\[
V_n = \frac{\partial \psi}{\partial n} = \frac{\partial \psi}{\partial x} l_x + \frac{\partial \psi}{\partial y} l_y = V_0 \quad \text{on} S_2 \quad (15)
\]

Where \( V_0 \) is the velocity normal to the boundary surface and \( l_x \) and \( l_y \) are the direction cosines.

The approximate solution for \( \Psi \) in the problem domain is given by

\[
\Psi = \sum_{i=1}^{n} [N_i^e] \{\Psi_i^e\} \quad (16)
\]

Where \( n \) is the total number of elements in the problem domain, \([N_i^e]\), is the element shape function matrix and \( \{\Psi_i^e\} \) is the vector matrix of nodal values of the stream function. Applying the principle in which the Galerkin shape function is taken as the weighted residual function and integrated over the domain \( S \).
\[ \int_{S} W_{j}R ds = 0 \] (17)

\[ \int_{S^{e}} \int N_{j}^{e} F(\psi_{a}^{e}) dS = 0 \] (18)

Where \( \psi_{a}^{e} \) is an approximate solution of the field variable \( \Psi \) in the element and \( S^{e} \) is the area of the element. \( \psi_{a}^{e} \) may be found by substituting the differential Eq. (13) into Eq. (15). Then by substitution into Eq. (18), we obtain:

\[ \int_{S^{e}} \left[ N_{j}^{e} \frac{1}{b} \frac{\partial^{2}}{\partial \kappa^{2}} \sum_{i}^{1} N_{i} \psi_{i} + \frac{1}{b} \frac{\partial^{2}}{\partial \gamma^{2}} \sum_{i}^{1} N_{i} \psi_{i} \right] dS = 0 \] (19)

Which after introduction of the boundary conditions may be expressed in matrix form:

\[ \sum_{i}^{n} [K^{e}]_{i} \{\psi\} = R \] (20)

Where \( [K^{e}] \) representing the element matrix. From assemblage, the final system becomes;

\[ [K] \{\Psi\} = R \] (21)

Where \( [K] \) being the global matrix for the whole domain and \( R \) to be evaluated at the domain boundaries AB and CD (fig. 1).

From experience, isoparametric element is found to give satisfactory results in computation of fluid flow dynamics. It uses the principle of applying the same shape
function to define the element shape as well as the field variable, using a two
dimensional quadrilateral finite elements (Fig. 3) with shape functions with respect
to local coordinates ζ and η, then the final form of the field variable matrix becomes

\[
\begin{bmatrix}
\frac{\partial \psi}{\partial x} \\
\frac{\partial \psi}{\partial y}
\end{bmatrix} = [J]^{-1}
\begin{bmatrix}
\frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \frac{\partial N_3}{\partial \zeta} & \frac{\partial N_4}{\partial \zeta} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta}
\end{bmatrix}
\begin{bmatrix}
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_4
\end{bmatrix}
\]  

(22)

With [J] being the Jacobean matrix for the isoparametric element in the x-y
plane.

Once the field variable is known, the velocities u and v at any point in the
field domain are obtained from the application of Eq. (10) and the resultant or
relative velocity can be obtained as:

\[
w = \sqrt{u^2 - v^2}
\]  

(23)

The cavitation number may be computed at the point of highest velocity at the
cavity wall (\(V_c\))

4- Discussion

\[
\sigma_b = \frac{V_c^2}{W_1^2} - 1
\]  

(24)

In the present paper, the stream function around a flat blade cascade of
hydrofoil has been computed over a range of blade angles and stagger angles. The
finite element method was applied for the solution of the Laplace equation for two-
dimensional, inviscid and irrotational flow. A computer program using FORTRAN
V was set up for this purpose (Abdul Wahab, M. (1995)), based on an Algorithm for flow around a cylinder in confined configuration (Chung ,T,J. (1978)) . The discretization of the domain into finite elements (sub regions) was done by using quadrilateral isoparametric elements. The size of the element near the blade tip and around the cavity has to be reduced considerably in order to obtain reasonable accuracy, (Fig. 4)

For incipient cavitation, the estimation of the cavitation number according to Eq. (24) necessitates the determination of the minimum pressure area in the channel corresponding to the highest velocity \( V_c \) at which the local pressure approaches the vapor pressure of the liquid . This normally occurs at the suction side of the blade which again is a function of the blade angle, the flow incidence angle and the solidity of the channel.

For developed cavitation, the choice of the type and size of cavity build up on the suction side had to be assumed. For this purpose a fixed cavity extending to about half the chord length with an average thickness varied between 10 and 50% of the channel width (pitch). The maximum value of the flow velocity (\( V_c \)) measured on the surface of the cavity was found to be at about 15-20% of cavity length from the leading edge because the flow in this region attains its highest curvature due to sudden change of direction.

Fig. (5), shows the plot for the theoretical values \( \sigma_b \) of for inception of cavitation as predicated by the finite element method for four different blade angle (\( \beta_1 = 15 - 30^\circ \)) with changing incidence angle (i) in the range -6 to 22°.

These results were compared to test data obtained from reference (9) (Pearsall,I. S. (1973)) on NACA 65-010 section for similar solidity (s=1). The comparison shows that the present theoretical data are in general lower than the experimental values, however they indicate similar trend. The lower values are believed to be due to idealization of the flow as depicted by the theoretical model.

Similar low values were obtained also by another recent work (Ardizon ,G. etal (1995)) who used a different theoretical for the estimation of based on geometrical and hydrodynamic analysis of the energy equation at blade entrance.

At developed cavitation, the cavitation number is quite high at the leading edge and decreases along the cavity. The effect of the incident angle (i) like before
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is to increase the cavitation number with increased \( i \), this is especially pronounced for small cavity size. In (Fig. 6) the emerging curves for as function of incidence angle is a clear cut performance for all 4 cavity sizes (i) incorporated in the finite element analysis. The representative \( \sigma_b \) for each cavity is chosen at the maximum velocity \( V_c \) at cavity boundary which was found to occur at relative cavity length \( L \) of. Results by other authors (Pearsall, J.S. (1973)) using a theoretical linearized potential theory for advanced cavitation is replotted on the same Fig. 6 showing good agreement with the present theory. Similar results were also obtained for developed cavitation applying theoretical approach (Kueny, J.L. et. al (1992)), for large attached cavities at impeller entrance.

Actual pump test data at advanced stage of cavitation with in the range 0.05-0.2 are within the data obtained for the theory. Since for the present model \( d_c / L \) corresponds nearly to the stepanoff’s B factor (Stepanoff,A.J. (1957)), earlier results have shown that for \( B=0.5 \) is reasonably good for developed cavitation in the range obtained for fig.6.

From Eq. (3), the optimum NPSE may be non-dimensionalized in the form

\[
\frac{\text{NPSE}}{\frac{1}{2} C_m^2} = A + \frac{B}{\phi}
\]

With \( A = 1 + \sigma_b \) and \( B = \sigma_b \). This relation is shown (Fig. 7) at different cavity size. The value of the relevant \( \sigma_b \) were extracted from fig.(6) at incident angle of 6\(^\circ\), which is the most widely expected one at normal operation. The results were compared on the same graph with result obtain from theory (Pearsall,J.S. (1973)) and those obtainable from actual pump tests (Wood, G.M. (1960)). Obviously, the results obtainable from the present theory compare well in the cavity size range applied with available information. The departure of the theory from real tests at height flow coefficient is of interest here, because it may involve the effect of instability and irrotation.

The suction specific speed is best related near inception point. In (Fig. 8), theoretical values for suction specific speed were plotted against flow coefficient Eq.
(7) by fixing the solidity, the incidence angle and diameter ratio at optimum values and varying the approach angle $\beta_1$ and the cavitation number $\sigma_b$.

The curves obtained show similar pattern for the 3 cases with NSS peaking up at mid values of $\phi$. Comparison to the results obtained from actual test results by NEL on a pump of similar geometry and replotted (Fig. 8) shows that theoretical prediction is close enough to real life test and may be used within good approximation.

Conclusion

In this paper an attempt was made to reach an easy (theoretical) method for the prediction of pumps suction parameter from available geometric and design data. The results are quite promising. They pave the way for further development of the theoretical model to include irrotation, and fluid compressibility; since the present theory gives optimistic values at high flow coefficients as compared to actual pump test.

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طريقة العناصر المحددة لتوقع أداء السحب

للضخات في حالة التجوف المتطور

إياد الجليبي

ملخص

في البحث الحالي اجريت محاولة لتطبيق طريقة العناصر المحددة لحل معادلة لابلس للجريان بطريقة الوضع وصلتها بطريق الجريان السائدة في مجرى محدد تعاليق (Cascade) والذي يمثل الجريان في مدخل البئرة للضخة الطاردة المركزية.

بإيجاد دالة خطيط لجريان (دالة الانسياب) في مواقع مختلفة داخل القناة أضخ بالإمكان الحصول نظرياً على قيمة ورقم التجوف في المناطق التي يصل فيها الضغط إلى أدنى مستوياته.

هذه الطريقة تجيز احتساب قيمة طاقة السحب للضغط الموجب والسعة النوعية للسحب للمضخة الناتجة في بداية وفي مراحل متقدمة من تكوين التجوف.

مقارنة النتائج المستخلصة من هذه النظرية مع نتائج مستخلصة من قبل باحثين آخرين نظريًا أعطت توافق جيد ضمن المجال المشمول بالبحث.

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Fig. (1) Cascade numerical solution domain

\[ \frac{\partial \psi}{\partial n} = V \]

\[ \psi = 0 \quad \psi = 0 \quad \psi = 0 \]

\[ \frac{\partial \psi}{\partial n} = V \]

Fig. (2) Cascade domain.
Fig. (3) Isoparametric quadrilateral element (Linear variation)

Fig. (4) Finite element domain discretization for developed cavity on suction side
Fig-5  Theoretical inception cavitation number as function of incidence angle and inlet blade angle at a solidity of

Fig-6  Minimum theoretical cavitation number at developed cavitation as function of incidence angle and cavity size.
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Fig. 7 Theoretical NPSE as function of flow coefficient and developed cavity size

Fig. 8 Theoretical NSS as function of flow coefficient and inlet blade angle for developed cavitation