Analyzing the Performance of Variable Sampling Interval Control Charts

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Abstract

Recently in the literature more work has been done on the modified control charts which are called the variable sampling interval control charts-VSI charts. One of the assumptions which has been analyzed by researchers is that the shift in process mean is constant. This assumption has been examined experimentally, using certain real data, in the first part of this work. In the second part, however, the shift assumed as a linear time series function, as a random variable of specific distribution such as the exponential or normal has been investigated. In this part the analysis has been exercised using the simulation technique as a tool.

In both cases an evaluation process in a form of comparison between the VSI charts and the traditional fixed sampling interval (FSI) charts have been adopted.

Throughout the analysis a parameter called the average time to signal-ATS is used as a statistical criterion for the purpose of comparison and evaluation. It has been concluded, using this criterion, that the VSI charts are proven to be superior to the FSI control charts.

1. Introduction

One of the common features of Shewhart Control Charts is that the time between samples taken from the process is constant (Runger, 1990). Recently however, there has been extensive development in the control charts area where sampling interval varies depending on what is observed from the process.

Charts utilizing this idea are called variable sampling interval- VSI control charts. The basic idea behind these charts is that after a sample is taken the time interval until the next sample should be short if there is some indication of process change and long if there is no indication on such change (Reynolds et al. 1988).

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In this paper these VSI charts are investigated and evaluated based on some assumptions regarding the shift in process mean. Experimental data have been analyzed for this purpose. Besides, theoretical analysis, using the simulation technique as a tool, to investigate the performance of these charts and compare it to the performance of the traditional fixed sampling interval – FSI charts has also been adopted.

2. Control Charts Performance

Many researchers, for example Amin and Miller (1993), Baxley (1995) and Reynolds et al. (1996) have supported the belief that the VSI charts are useful and could be more efficient as compared to the traditional charts.

To measure the usefulness of these charts, their performances have been compared here to that of the FSI charts. The performance analysis has been exercised in this research on the $\bar{X}$- charts. VSI and FSI $\bar{X}$- charts have been evaluated. The evaluation criteria are parameters like the average time to signal – ATS, and the average number of samples to signal – ANSS. “To signal” here means until the detection of shift in process mean.

In fact if both charts – the VSI and the FSI charts are setup at the same control limits and if no shift has occurred in process mean ($\mu = \mu_0$) then the VSI chart will match the FSI chart in the sense that they both require the same number of sample to signal ( Reynolds et al. 1988 ) and the same average sampling rate and false alarm rate over time. However, if process mean has shifted to $\mu_1$ VSI and FSI charts can be compared to determine which chart will do better in detecting the shift in $\mu$.

3. Charts Construction

As in any quality control experiment, certain quality characteristic has to be identified before data are collected for the purpose of construction the corresponding control chart. For the purpose of this work certain “pipe diameter” is used as a quality characteristic and the summary for the $\bar{X}$ and R (range) values for 29 samples obtained are summarized in table(1).

Charts to be constructed here are the $\bar{X}$-charts used to control process central tendency and the corresponding R-chart which is necessary to control process variability.

Now using data of table (1):

$$\bar{R} = \frac{\sum_{i=1}^{29} R_i}{29} = 2.997598 \approx 0.1034$$
Analyzing the Performance of Variable Sampling Interval Control Charts

And using the range method (Duncan 1974) with n=8 and d_2=2.847 (d_2 is a factor obtained from special table with n=8) the process standard deviation can be determined using:

\[ \hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.1034}{2.847} = 0.0363 \]

Applying these process parameters the X-bar chart and the R-chart have been constructed (figures 1 and 2)

4. Parameters Design of the VSI and FSI charts

The standard one hour interval will be used for the FSI three sigma control chart. To determine the intervals for the VSI charts we have to look at table (1) which shows that a maximum sampling time of 29 minutes is needed. This suggests that 30 minutes (or 0.5 hour) is the minimum possible sampling interval necessary for the VSI charts.

In this study three VSI charts will be examined and evaluated against the FSI chart. Two of these charts have two sampling intervals (\( \eta = 2 \)) while the third chart will be of three sampling intervals (\( \eta = 3 \)).

The two interval charts have different spacing. These charts are of \( 0.5 \) hr, \( 1.5 \) hr \) and \( 0.75 \) hr, \( 1.25 \) hrs \). The third VSI chart, however, has sampling interval of \( 0.5, 1, 1.5 \). These charts, as can be noted, have sampling intervals symmetric around the fixed sampling interval of one hour.

As mentioned earlier, the ATS is one of the important parameters that can be used for the evaluation purpose of these charts. This parameter has been used for this purpose by authors such as Prabhu et al. (1995). It is also used, here in the experimental part of this research. Accordingly the ATS for the FSI chart and the three types of the VSI charts, described above, have been determined experimentally and results obtained are depicted in table (2).

The table shows time to signal and number of samples needed to signal at the state of statistical control for all the charts. Thus a matching condition between all charts exist. Results of these parameters in this table explain that two of the VSI charts perform better, based on their time to signal, than the FSI chart. However, to support this conclusion, more work has to be done. This would justify the second part of this work, namely the simulation analysis of the above-mentioned VSI and FSI control charts.

5. Reexamining the ATS Property of the charts

When examining the time to signal property in the previous section, nothing has been said about the distribution of process shift. Since this shift may not be
considered constant over time, this section will focus on this point, and charts will be examined at different distributions of that shift.

Theoretical analysis, utilizing the simulation technique as a tool, is undertaken here. The technique will be used to estimate parameters needed for the purpose of performance comparison between FSI and different styles of VSI charts. Within this evaluation process, the spacing of sampling intervals and its effect on chart efficiency, the distribution of time at which process stays in the state of statistical control will also be examined.

Three cases of process shift will be considered. First, process shift is taken as a linear time series function. The second case would assume the shift as exponentially distributed random variable, while the third would examine the shift as a normal random variable.

5.1. Process Shift Values as Linear Time Series Function

In this model it is assumed that the $\bar{X}$-chart is used to control the process of normal observations with mean $= \mu$ and variance $\sigma^2$. When sampling from this process, sample means are computed and plotted on the chart. Then:

(i) If sample mean falls outside control limits, then signal;
(ii) If USLA $\leq$ sample mean $< \text{UCL}$ or
LCL $\leq$ sample mean $< \text{LSLA}$ then take next sample after $d_1$
(iii) If no one of the above, then take next sample after $d_2$

Where:

UCL and LCL: upper and lower control limits of the chart
USLA and LSLA: upper and lower sectional limits located inside the
UCL and LCL respectively

d1, d2: are sampling intervals where d1 < d2

Furthermore, the model assumes that time at which shifts occur is exponential with mean equals to $\lambda^{-1}$. Also, when shift occurs it will be changed over time (shift amount) according to the linear time series function.

Now let the following function represent the time series of shift amount

$$\delta_t = \delta_{in} + \theta(t - t_s) \text{ for } t \geq t_s$$

$$\delta_t = 0 \text{ for } t < t_s$$

Where:

$\delta_t$: shift amount at time $t$
\[ \delta_n = \text{initial value of shift at time } t, \]

\[ \theta = \text{Slope of shift line} \]

Since shift would occur at random times and since sampling interval is not constant then the parameter "Adjusted Average Time to Signal" - AATS will be used as a measure of chart efficiency. This parameter will be adopted in this simulation study.

The Beginners All purpose Symbolic Instruction Code (MS-BASIC) has been adopted to be used as a simulation program for the purpose of analyzing this model. This program, which is depicted in Appendices A and B (depending on shift pattern) is designed to enable the user to specify all chart parameters such as number of standard deviations (\( h \)), sample size (\( n \)), the fixed sampling interval (FSI), the variable sampling intervals (\( d_1 \) and \( d_2 \))..etc. Besides, shift equation parameters can also be specified.

As all these input data are entered the program, at this point, would determine control limits and sectional limits and become ready to execute the simulation process. The program would treat the sample averages as normal random variables generated by the direct transformation technique. This technique would also be used by the program to generate the exponentially distributed random variable representing shift times. The program would also use the shift equation to determine the shift amount and so forth.

For the purpose of using the program the traditional FSI chart will be considered as a special case of the VSI charts of one sampling interval equals to \( d_1 + d_2 \).

The number of simulation runs used throughout the research was 2000. This number can be justified as follows. Usually the number is determined such that certain estimation accuracy (\( \epsilon \)) of the simulation process is stated. Moreover, that accuracy is desired to be attained with certain probability (1-\( \alpha \)). A formula taking these considerations into account is given in Banks and Carson (1984) as follow:

\[
\text{Number of runs } \geq \left( \frac{Z_{\alpha/2} \cdot S}{\epsilon} \right)^2
\]

Where \( S \) is the standard deviation of some preliminary number of runs. Ten runs are first executed for this purpose (for one of the cases of Table (3) with \( \theta = 0.005 \)) which provide an S value for the AATS of 0.4476. Now, if the desired accuracy level is \( \pm 0.02 \) and if (1-\( \alpha \)) is 0.95 then the number of runs would be greater than or equal 1924. Hence, 2000 runs are more than enough.
As for the simulation output the AATS and the average number of samples to signal are the main two program outputs. To get these outputs the program is designed to run for 15 run lengths which can all be specified by the user. The adjusted time to signal for each run is estimated and then the average of these times is determined for all the runs.

Now, to understand the simulation process, assume that process starts with mean equal to \( \mu_o \) which then shifts to \( \mu_1 \) at some random point in future time. The adjusted time to signal determined by the model is the sum of times \( Y \) and \( Q \) where \( Y \) is the time from shift to next sample while \( Q \) is the time from next sample after shift to signal — when sample point falls outside control limits.

Comparison between behavior of FSI and VSI (\( \eta = 2 \)) charts is shown in table (3). The table indicates that at small shift levels (6) AATS of the VSI charts are less than that of the FSI chart especially when the two sampling intervals are spaced far apart — the case of (0.1, 1.9 hr). The sampling costs however — indicated by ANSS are relatively higher in the case of the VSI charts.

On the other hand and for moderate and large shift levels FSI charts has slightly greater AATS even though its Q interval is the smallest. This is in part due to the fact that when shifts are large they are detected after very short time and that may happen after taking the first sample after shift. In this case the AATS may equal Y interval only where the Q interval may equal zero. But in general we can conclude that for (\( d_1, d_2 \) = (0.1, 1.9), (0.5, 1.5)) VSI charts these charts have better performance over a wide range of shifts as compared to the equivalent FSI chart.

Let us now examine table (4) where the mean value of shift time is relatively very large \( \lambda^{-1} = 100 \) and other parameters are kept the same. In this case, \( Y_{av} \). Values are different for all charts as expected. Besides, the AATS of the VSI charts are still better than those of the FSI charts even though the FSI chart has small Q values at large shifts. For very large shift values detection may occur by the first sample which causes the AATS to equal \( Y_{av} \). Then charts with minimum \( Y_{av} \) become faster in shift detection. In this case the FSI chart may perform better in detecting shifts.

5.2. Process Shift Values as Exponentially Distributed Random Variable

Due to the fact that assignable causes, such as tool wear, may increase with time, shift amount pattern as positive exponentially distributed becomes reasonable. In this model, of course, both the amount of shift and the timing of shift are exponential.

Assume small mean time of shift occurrence, \( \lambda^{-1} = 0.00001 \) hr and let \( \eta=2 \). Also assume that sampling intervals are symmetric about the fixed value of \( d = 1 \) hr, and equal interval probabilities at the state of statistical control. Table (5) represents this model with \( h=3 \) and \( n = 5 \). This table demonstrates that at all shift values \( (\lambda d^1) \) the AATS values of the VSI charts are less (better) than that of the traditional FSI chart. This is especially true when the two sampling intervals are paced far apart.
Table (6) shows simulation results for the same model except that $\lambda = 100$hr or relatively large mean for shift times. Once again except for very large shifts the VSI charts perform better than the FSI charts. However, if $Q_{av}$ is used as a comparison criterion we note that VSI chart would perform better in all process shift levels.

5.3 Process Shift Amount as Normally Distributed Random Variable

In this model, amount of process shift is assumed to be normally distributed random variable with parameters $\mu_0$ and $\sigma_0$ representing the mean and standard deviation.

Tables (7) and (8) demonstrate the simulation results of models having $\eta = 2$, $\eta = 4$ but one with $\lambda^{-1} = 0.0001$ hr and the other with $\lambda^{-1} = 100$hr. Again, these tables show that the VSI charts work better than the FSI chart at most shift values ($\mu_0$) except for very large shift value.

5.4 Statistical Tests

To support the findings explained in sections 5.1, 5.2, and 5.3 regarding the superiority of the VSI charts over the FSI charts, more statistical tests have been made.

Since ATS and AATS are the main parameters used, for the purpose of the comparison, tests concerning these parameters are considered. As the parameters represent mean values, tests namely hypotheses tests on two means are useful here. Such tests are available in most of the statistical texts such as that by Walpole and Myers (1989). The test is summarized as follows.

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_1: \mu_1 - \mu_2 \neq 0$$

Where $\mu_1$ may represent the population ATS and $\mu_2$ may represent the population AATS. Now, if $H_0$ is accepted it means there is no significance difference between values of ATS and AATS, otherwise there is such difference.

The test statistic is the calculated $t$-value. To calculate this value, sample standard deviations for both ATS and AATS ($s_1^2$ and $s_2^2$) are needed since no information available on the standard deviation of their population. Fortunately, these values are available since 15 groups have been examined as explain in the paper. As a result of these groups mean values (ATS and AATS) are obtained as depicted in tables (3) through (8).

Now, the calculated $t$ value =

$$\frac{(\bar{x}_1 - \bar{x}_2) - d_0}{S_{p} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where $\bar{x}_1$ = sample ATS, and $\bar{x}_2$ = sample AATS.
and \( \mu_0 = \mu_1 - \mu_2 = 0 \) in our case

\[ n_1 = n_2 = 15 \text{ in our case} \]

And \( S_p = \text{pooled standard deviation} \)

\[ S_p^2 = \frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{(n_1 + n_2) - 2} \]

To apply this test on the first case of table (3), for example, where ATS = 89.76 and AATS = 83.27:

\[ S_p^2 = \frac{0.917(14) + 0.805(14)}{28} = 0.861, \quad \text{or} \quad S_p = 0.928 \]

and calculated \( t \)-value = \( \frac{(89.76 - 83.27) - 0}{0.928} \sqrt{\frac{1}{15} + \frac{1}{15}} = 19.15 \)

And from the well-known \( t \)-table we get the tabulated \( t \)-value that corresponds to significance level (1 - \( \alpha \)) of 0.95 to be 2.048 [area of 0.025 and degrees of freedom of \((n_1+n_2)-2 = 28\)]. Now, since 19.15 > 2.048, we reject \( H_0 \) and conclude that the difference between the ATS and AATS is significantly greater than zero.

Same procedures are applied to the rest of the cases in tables (3) through (8), and some other \( t \)-values are shown in table (9). These \( t \)-values are also compared to the tabulated \( t \)-value of 2.048. Examining table (9) reveals that all of the results, except the last one, have calculated \( t \)-values > 2.048, and that lead to the rejection of the null hypothesis and to the admission that a significant difference between the means (ATS and AATS) exists. In other words, we have proven with certain statistical significance that the ATS is greater than the AATS of the variable sampling interval charts which makes the later more desirable and efficient ones.

6. Conclusions

a. When analyzing experimental data, assuming that shifts are kept constant in magnitude (no specific pattern), results have showed that VSI charts are superior to the FSI charts in detecting process shift. This means that the time to signal in case of the VSI charts is relatively smaller.

b. Simulation results and the following statistical tests, when process shift modeled as linear time series function, have demonstrated that the VSI charts perform better in detecting shifts than the FSI charts, especially when process shifts are relatively small.

c. Simulation results and the following statistical tests have also demonstrated, when shift amounts are distributed normally or
Analyzing the Performance of Variable Sampling Interval Control Charts

exponentially, that VSI charts are again superior to the FSI charts in detecting shifts especially when sampling intervals are spaced far apart.

تحلیل اداء لوحات السيطرة ذات الفترات المتغيرة بين أخذ العينات

عبدالرسول الحياني و جلال الفزيري

ملخص

تناولت البحوث في مجال السيطرة النوعية (ضبط الجودة) في الأولى الأخيرة موضوع لوحات السيطرة المحورية التي تسمى لوحات السيطرة ذات الفترات الزمنية المتغيرة بين فحص العينات المختلفة. لقد تعالجت الكثير من هذه البحوث مع الافتراض القائل أن الانحرافات العاصلة لمتوسط العمليات الانتاجية هي انحرافات ذات قيم ثابتة مع الوقت.

لقد تم في الجزء الأول من هذا البحث والاستنبذ إلى الافتراض إعلاه دراسة عملية انتباهية حقيقية لفرض تقييم النوع المذكور إعلاه من لوحات السيطرة. أما الجزء الثاني من هذا العمل فقد تركز حول الافتراض بإمكانية أن يكون الانحراف على شكل دالة رياضية خطية متغيرة مع الزمن تارة وعلي شكل متغير عشوائي ذا توزيع إحصائي معين مثل التوزيع الطبيعي أو الامثا تارة أخرى. في هذا الجزء الأخير تم استخدام أسلوب المحاكاة كدراسة للتحلیل.

وفي الحالات أجريت عمليات تقييم ومقارنة بين لوحات السيطرة المحورية واللوحات التقليدية ذات الفترات ثابتة. وخلال هذه العمليات تم اعتماد ما يسمى بمتوسط الوقت اللازم لاستكشاف الانحرافات الخاص بالعملية كمقياس إحصائي لافتراض المقارنات إعلاه.

وقد تم التوصل إلى استنتاج يؤكد تفوق اللوحات المحورية على التقليدية في سرعة اكتشافها للانحرافات بموجب المعيار المذكور.
References


Reynolds, M.R., $\bar{X}$ - charts with Variable Sampling Intervals, Technometrics, 30 (1988) 181-192


Analyzing the Performance of Variable Sampling Interval Control Charts

**Figure (1)**

The FSI $\overline{X}$ - Chart

**Figure (2)**

The FSI $R$ - Chart
### Table (1)

The Observation Sheet and Samples Statistics

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>X-bar</th>
<th>R</th>
<th>Time Spent in Sampling (min)</th>
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<td>1</td>
<td>27.3594</td>
<td>0.1625</td>
<td>25</td>
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<tr>
<td>2</td>
<td>27.3153</td>
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<td>3</td>
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<td>0.1500</td>
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<td>0.1000</td>
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</tr>
<tr>
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<td>0.1500</td>
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</tr>
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<td>27.2968</td>
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</table>
Analyzing the Performance of Variable Sampling Interval Control Charts

Table (2)

<table>
<thead>
<tr>
<th>Chart type</th>
<th>Sampling Intervals</th>
<th>Time to Signal (min)</th>
<th>No. of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>FS</td>
<td>1 - hour</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>VSI</td>
<td>(0.5, 1.5)</td>
<td>38.5</td>
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</tr>
<tr>
<td>VSI</td>
<td>(0.75, 1.25)</td>
<td>39</td>
<td>40</td>
</tr>
<tr>
<td>VSI</td>
<td>(0.5, 1.5)</td>
<td>38.5</td>
<td>38</td>
</tr>
</tbody>
</table>

Table (3)

Simulated AATS for $\lambda^{-1} = 0.00001$, h=3, n=4. The Shift is Linear Time Series Function, the First Sample Taken after $d_1$.

<table>
<thead>
<tr>
<th>0</th>
<th>Fixed Interval ($Y_{\text{av}}=0.99$)</th>
<th>Variable Sampling Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{av}}$</td>
<td>ATS</td>
</tr>
<tr>
<td>0.005</td>
<td>88.76</td>
<td>89.76</td>
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<tr>
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<td>1.00</td>
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<td>1.98</td>
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<td>2.00</td>
<td>0.16</td>
<td>1.16</td>
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<tr>
<td>$\infty$</td>
<td>0.00</td>
<td>0.99</td>
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Table (4)
Simulated AATS for $\lambda = 100$, $h=3$, $n=4$. The Shift is Linear Time Series Function, the First Sample Taken after $d_1$

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>Fixed Interval ($Y_{av}=0.99$)</th>
<th>Variable Sampling Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{av}$</td>
<td>ATS</td>
</tr>
<tr>
<td>0.005</td>
<td>89.33</td>
<td>89.85</td>
</tr>
<tr>
<td>0.050</td>
<td>17.78</td>
<td>18.29</td>
</tr>
<tr>
<td>0.50</td>
<td>2.77</td>
<td>3.28</td>
</tr>
<tr>
<td>1.00</td>
<td>1.45</td>
<td>1.97</td>
</tr>
<tr>
<td>2.00</td>
<td>0.73</td>
<td>1.25</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.00</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table (5)
Simulated AATS for $\lambda = 0.00001$, $h=3$, $n=5$. When the Process Shift is Positive Exponential Random Variable and the First Sample Taken after $d_1$

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>Fixed Interval ($Y_{av}=0.99$)</th>
<th>Variable Sampling Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{av}$</td>
<td>ATS</td>
</tr>
<tr>
<td>0.25</td>
<td>43.57</td>
<td>43.57</td>
</tr>
<tr>
<td>0.50</td>
<td>8.76</td>
<td>9.76</td>
</tr>
<tr>
<td>0.75</td>
<td>4.01</td>
<td>5.01</td>
</tr>
<tr>
<td>1.00</td>
<td>2.42</td>
<td>3.42</td>
</tr>
<tr>
<td>1.25</td>
<td>1.78</td>
<td>2.78</td>
</tr>
<tr>
<td>2.00</td>
<td>0.89</td>
<td>1.89</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Analyzing the Performance of Variable Sampling Interval Control Charts

Table (6)
Simulated AATS for $\lambda^{-1} = 100$, $h=3$, $n=5$. The Shift is Linear Time Series Functions, the First Sample Taken after $d_1$

<table>
<thead>
<tr>
<th>$\lambda^{-1}$</th>
<th>Fixed Interval ($Y_{av}=0.52$)</th>
<th>Variable Sampling Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{av}$</td>
<td>ATS</td>
</tr>
<tr>
<td>0.25</td>
<td>43.67</td>
<td>44.19</td>
</tr>
<tr>
<td>0.50</td>
<td>8.90</td>
<td>9.42</td>
</tr>
<tr>
<td>0.75</td>
<td>4.03</td>
<td>4.35</td>
</tr>
<tr>
<td>1.00</td>
<td>2.53</td>
<td>3.05</td>
</tr>
<tr>
<td>1.25</td>
<td>1.78</td>
<td>2.80</td>
</tr>
<tr>
<td>2.00</td>
<td>0.91</td>
<td>1.43</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.00</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table (7)
Simulated AATS for Symmetric VSI chart with $\lambda^{-1} = 0.00001$, $h = 3$, $n = 4$. Shift is Normally Distributed Random Variable $\sigma_x = 0.50$

<table>
<thead>
<tr>
<th>$\mu_x$</th>
<th>Fixed Interval ($Y_{av}=0.99$)</th>
<th>Variable Sampling Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{av}$</td>
<td>ATS</td>
</tr>
<tr>
<td>0.25</td>
<td>21.12</td>
<td>22.12</td>
</tr>
<tr>
<td>0.45</td>
<td>13.09</td>
<td>14.09</td>
</tr>
<tr>
<td>0.75</td>
<td>5.87</td>
<td>6.87</td>
</tr>
<tr>
<td>1.00</td>
<td>3.14</td>
<td>4.14</td>
</tr>
<tr>
<td>2.00</td>
<td>0.32</td>
<td>1.32</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>
### Table (8)

Simulated AATS for $\lambda = 100$, $h = 3$, $n = 4$, Normally Distributed Shift, with $\sigma = 0.5$

<table>
<thead>
<tr>
<th>$\mu_*$</th>
<th>Fixed Interval ($Y_{av} = 0.52$)</th>
<th>Variable Sampling Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{av}$</td>
<td>AATS</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.45</td>
<td>13.03</td>
<td>13.55</td>
</tr>
<tr>
<td>0.75</td>
<td>5.89</td>
<td>6.40</td>
</tr>
<tr>
<td>1.00</td>
<td>3.21</td>
<td>3.72</td>
</tr>
<tr>
<td>2.00</td>
<td>0.31</td>
<td>0.82</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.00</td>
<td>0.52</td>
</tr>
</tbody>
</table>

### Table (9)

The Calculated $t$ Values

<table>
<thead>
<tr>
<th>$ATS$</th>
<th>$AATS$</th>
<th>Taking from Table</th>
<th>$Sp$</th>
<th>$t$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.37</td>
<td>15.89</td>
<td>3</td>
<td>0.3111</td>
<td>21.83 &gt; 2.048</td>
</tr>
<tr>
<td>3.280</td>
<td>2.670</td>
<td>3</td>
<td>0.0226</td>
<td>64.28 &gt; 2.048</td>
</tr>
<tr>
<td>89.85</td>
<td>83.50</td>
<td>4</td>
<td>0.3840</td>
<td>45.19 &gt; 2.048</td>
</tr>
<tr>
<td>18.29</td>
<td>15.87</td>
<td>4</td>
<td>0.3291</td>
<td>20.14 &gt; 2.048</td>
</tr>
<tr>
<td>3.280</td>
<td>2.800</td>
<td>4</td>
<td>0.0712</td>
<td>18.47 &gt; 2.048</td>
</tr>
<tr>
<td>1.970</td>
<td>1.300</td>
<td>4</td>
<td>0.0592</td>
<td>31.01 &gt; 2.048</td>
</tr>
<tr>
<td>43.57</td>
<td>37.99</td>
<td>5</td>
<td>0.3034</td>
<td>30.39 &gt; 2.048</td>
</tr>
<tr>
<td>9.760</td>
<td>6.630</td>
<td>5</td>
<td>0.2478</td>
<td>34.61 &gt; 2.048</td>
</tr>
<tr>
<td>5.010</td>
<td>2.810</td>
<td>5</td>
<td>0.1971</td>
<td>35.25 &gt; 2.048</td>
</tr>
<tr>
<td>3.420</td>
<td>1.650</td>
<td>5</td>
<td>0.0932</td>
<td>52.03 &gt; 2.048</td>
</tr>
<tr>
<td>2.780</td>
<td>1.170</td>
<td>5</td>
<td>0.0652</td>
<td>67.65 &gt; 2.048</td>
</tr>
<tr>
<td>1.890</td>
<td>0.620</td>
<td>5</td>
<td>0.0442</td>
<td>78.72 &gt; 2.048</td>
</tr>
</tbody>
</table>
## Analyzing the Performance of Variable Sampling Interval Control Charts

**Cont. Table (9)**

<table>
<thead>
<tr>
<th>ATS</th>
<th>AATS</th>
<th>Taking from Table</th>
<th>Sp</th>
<th>t – value</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.19</td>
<td>38.39</td>
<td>6</td>
<td>0.3551</td>
<td>40.12 &gt; 2.048</td>
</tr>
<tr>
<td>9.420</td>
<td>7.450</td>
<td>6</td>
<td>0.3064</td>
<td>14.31 &gt; 2.048</td>
</tr>
<tr>
<td>4.350</td>
<td>3.630</td>
<td>6</td>
<td>0.0592</td>
<td>18.71 &gt; 2.048</td>
</tr>
<tr>
<td>3.050</td>
<td>2.490</td>
<td>6</td>
<td>0.1018</td>
<td>15.07 &gt; 2.048</td>
</tr>
<tr>
<td>2.800</td>
<td>2.000</td>
<td>6</td>
<td>0.1422</td>
<td>15.41 &gt; 2.048</td>
</tr>
<tr>
<td>22.12</td>
<td>18.69</td>
<td>7</td>
<td>0.6667</td>
<td>14.10 &gt; 2.048</td>
</tr>
<tr>
<td>14.09</td>
<td>11.19</td>
<td>7</td>
<td>0.7166</td>
<td>11.09 &gt; 2.048</td>
</tr>
<tr>
<td>6.870</td>
<td>4.920</td>
<td>7</td>
<td>0.5745</td>
<td>9.300 &gt; 2.048</td>
</tr>
<tr>
<td>4.140</td>
<td>2.670</td>
<td>7</td>
<td>0.2757</td>
<td>14.31 &gt; 2.048</td>
</tr>
<tr>
<td>1.320</td>
<td>0.670</td>
<td>7</td>
<td>0.0866</td>
<td>20.56 &gt; 2.048</td>
</tr>
<tr>
<td>21.58</td>
<td>18.82</td>
<td>8</td>
<td>0.5756</td>
<td>13.14 &gt; 2.048</td>
</tr>
<tr>
<td>13.55</td>
<td>11.37</td>
<td>8</td>
<td>0.3108</td>
<td>19.22 &gt; 2.048</td>
</tr>
<tr>
<td>6.400</td>
<td>5.030</td>
<td>8</td>
<td>0.3092</td>
<td>12.14 &gt; 2.048</td>
</tr>
<tr>
<td>3.720</td>
<td>2.790</td>
<td>8</td>
<td>0.2649</td>
<td>9.620 &gt; 2.048</td>
</tr>
<tr>
<td>0.820</td>
<td>0.770</td>
<td>8</td>
<td>0.1217</td>
<td>1.126 &lt; 2.048</td>
</tr>
</tbody>
</table>
Appendix A

List of the Computer Program - The Time Series Case

10  KEY OFF
20  DIM OB (4000), ST (4300), D(4100), T(20), NA (20), P(20), ZA(20)
30  REM ****************************
40  REM * This computer program designed to simulate a process by *
50  REM * using the VSI control charts of two sampling interval, *
60  REM * where the shift is linearly increasing, and occur at a *
70  REM * random point of time in the future *
80  REM ****************************
90  REM
100 REM RABNDOMIZE (TIME R)
110 REM
120 REM * REM *
130 REM * h=the width of the control limits. *
140 REM * h1=The width of the sectional limit area. *
150 REM * h=N= the sample size *
160 REM * FSI=The fixed sampling interval *
170 REM * Lam=The average of shift occurrence *
180 REM * Slope = The shift increasing per unit time per process *
190 REM * Standard deviation *
200 REM * Delta() =The shift value at time 0 *
210 REM * Ob () =The shift value at time 0 *
220 REM * St () =The sampling time of sample number () *
230 REM * D1,D2=The minimum and maximum sampling intervals *
240 REM * P1,p2=The sampling intervals probabilities *
250 REM ****************************
260 REM
270 REM
280 REM
290 REM ****************************
300 REM * Entering the chart parameters *
Analyzing the Performance of Variable Sampling Interval Control Charts

REM ***************************************************************
CLS: LOCATE 8,20: INPUT "Enter the width of the control limits": H
LOCATE 9,20: INPUT "Enter the sample size": N
LOCATE 10,20: INPUT "Enter the fixed sampling interval": FSI
LOCATE 11,20: INPUT "Enter the min. sampling interval": D1
LOCATE 12,20: INPUT "Enter the max. sampling interval": D2
LOCATE 13,20: INPUT "Enter the interval of first sample": FD
LOCATE 14,20: INPUT "Enter the average of shift occurrence": LAM
LOCATE 15,20: INPUT "Enter the slope of the shift line": SLOPE
LOCATE 16,20: INPUT "Enter initial shift value": DELTA0
LOCATE 17,20: INPUT "Enter the how many times to run each group": CIR
REM ***************************************************************
REM
REM
REM ***************************************************************
REM * Calculation of q value at the specified width of control limits. Using formula 26.2.17 in Abramowitz and Stegun
REM * (1965), Handbook of Mathematical Functions. Dover
REM * Publication, Inc.; NY
REM ***************************************************************
A1=.31938153
A2=.356563782#
A3=1.781477937#
A4=-1.821255978#
A5=.330274429#
R=.2316419
PIE=3.1415927#
IF H<0 THEN LET M1=-H ELSE M1=H
D=1/(1 + (R*M1))
PTVAL=(1/PIE) * (EXP(-.5*M1^2))
IF H<0 THEN LET P=1-P
Q=(1-P)*2
REM *****************************************************************
IF D1=D2 THEN 670 ELSE P1=((D2-FSI)/(D2-D1))*(1-Q): P2=1-Q-P1
GOTO 680
P1=(1-Q)/2 : P3 = P1
REM *****************************************************************
REM *****************************************************************
REM * COMPUTING number of sigma for the sectional limit area *
REM * using the inverse normal CDF formula 26.2.23 in abramowitz *
REM * and stegun (1965) . Handbook of Mathematical Functions, Dover *
REM * Publicationms, Inc., NV. *
REM *****************************************************************
P=1-(P1/2)-(Q/2): PROB=P
IF P>=1 THEN 800 ELSE 820
H1=1
GOTO 970
IF P<=0 THEN 830 ELSE 850
H1=-1
GOTO 970
IF P>.5 THEN P=1-P
G=1/(P^2)
G=LOG (G)
G=SQR (G)
XP=1+1.432788*G+.189269*(G^2)+.0010308*(G^3)
XP=(2.515517+(.802853*G)+(.010328*(G^2)))/XP
XP = XP - G
IF PROB>.5 THEN XP=-XP
Analyzing the Performance of Variable Sampling Interval Control Charts

930     H1=XP
940     REM *******************************************************************
950     REM *  Calculation of the control limits *
960     REM *******************************************************************
970     MEAN=0:SIGMA=1:SIGMAX=SIGMA/(SQR(N))
980     UCL=MEAN+H*SIGMAX
990     CL=MEAN
1000    LCL=MEAN-H*SIGMAX
1010    USLA=MEAN+H1*SIGMAX
1020    LSLA=MEAN-H1*SIGMAX
1030     REM *******************************************************************
1040     REM
1050     REM
1060     REM
1070     REM *******************************************************************
1080     REM *  Printing the output of chart parameters entered before *
1090     REM *******************************************************************
1100     CLS
1110     PRINT " Simulation Inputs : using two (SVSI) control chart "
1120     PRINT "Matched for FSI control chart, the shift is linearly drifts";
1130     PRINT "and occurs as an exponential random variable in the future";
1140     PRINT
1150     PRINT "The width of control limits is";H;"process sigma"
1160     PRINT "The width of sectional limits area";H1;"process sigma"
1170     PRINT " The sample size is"; N
1180     PRINT " The fixed sampling interval is"; FSI; "hour"
1190     PRINT "The upper control limit is"; UCL
1200     PRINT "The lower control limits is"; LCL
1210     PRINT "The upper sectional limit area is"; USLA
1220     PRINT "The lower sectional limit area is"; LSLA
1230     PRINT "The center line is"; CL
1240     PRINT "The minimum sampling interval is"; D1; "with" ;P1; "probability"
PRINT "The maximum sampling interval is " : D2: "with" : P2: "probability"
PRINT "The first sample will takes after " : FD
PRINT "The average time of shift occurrence is " : LAM
PRINT "The slope of the shift line is " : SLOPE
PRINT "The initial shift value is " : DELTA0
LOCATE 20,1:INPUT "Press enter to continue ":DS$
IF DS$="" THEN 1320 ELSE 1100
REM ******************************************************
REM * STARTING THE MONITORING PROCESS. *
REM ******************************************************
FOR LOOP1 = TO 15
FOR LOOP = 1 TO CIR
K1=RAN
TS0=(-LAM)*LOG(1-K1)
J=1
IF J=1 THEN 1410 ELSE 1420
ST(I)=FD
IF ST(I)<TS0 THEN 1430 ELSE 1450
D(I)=0
GOTO 1460
D(I) = DELTA0+SLOPE*(ST(I) - TS0)
MEAN1=MEAN+ D(I) * SIGMA
K2=RAD:K3=RND
Z=(-2*LOG(K2))^.5*SIN(2*3.14*K3)
IF D(I) = 0 THEN 1500 ELSE 1510
OB(I) = MEAN0+Z*SIGMAX:GOTO 1520
OB(I)=MEAN1+Z*SIGMAX
IF ST(I)<TS0 THEN J=J+1 ELSE J=0
IF J=1 THEN O=ST(I) - TS0 ELSE 1540
IF OB(I)>UCL OR OB(I)<=LCL THEN 1590 ELSE 1550
IF OB(I)<UCL AND OB(I)>=USL0 THEN 1570 ELSE 1560
IF OB(I)<LCL AND OB(I)<=LSL THEN 1570 ELSE 1580
Analyzing the Performance of Variable Sampling Interval Control Charts

1570  I=I+1:ST(I)=ST(I-1)+D1: GOTO 1400
1580  I=I+1:ST(I)=ST(I-1)+D2: GOTO 1400
1590  REM ***********************************************
1600  IF ST(I)<TS0 THEN 1570 ELSE 1610
1610  TIME=ST(I) - ST0
1620  ATS=ST(I) - TS0: O: CLS
1630  LOCATE 10.1: PRINT "GROUP#: LOOP1,"RUN#: LOOP
1640  PRINT "The adjusted time to signal is"; ST(I) - TS0
1650  PRINT "The number of samples taken after the shift occur are"; I
1660  PRINT "The time of shift starting is"; TS0
1670  PRINT "The chart signal at shift value equal"; D(I)
1680  PRINT "The time between the shift starting point and next sample is"; 0
1690  PRINT "The time from the first after the shift sample until the shift is"; ATS
1700  AVER=AVER+ST(I) - TS0
1710  SAMPLE=SAMPLE+J: FATRA=FTRA+O: AT+ATS
1720  J=0: NEXT
1730  LOOP:AVER=AVER/CIR: SAMPLE=SAMPLE/CIR: FTRA=FTRA/CIR: AT =AT/CIR
1740  AAT=AAT+AT: AATS=AATS+AVER: NSS=NSS+SAMPLE: AFTRA=AFTRA+(LOOP1)=AVER: NA(LOOP1)=SAMPLE: F(LOOP1)=FATRA: ZA(LOOP1)=AT: AVER=0: SAMPLE=0: FATRA=0: AT0
1750  NEXT LOOP: CLS: PLAY "d2"
1760  PRINT "Group AAT ANSS QAVR."
1770  PRINT "-----------------------------------------------"
1780  FOR I=1 TO 15
1790  PRINT I,T(I),NA(I),ZA(I)
1800  NEXT I
1810  PRINT "-----------------------------------------------"
1820  PRINT "Adjusted average time to signal is"; AATS/15
1830  PRINT "Average number of samples taken after the shift are"; SS/15
1840  PRINT "Average time between shift point and next sample is"; AFTRA/15
1850  PRINT "Average time to Starting from first sample after the shift is"; AAT/15

-435-
Appendix B

List of the Computer Program – The Exponential and Normal Case

10 KEY OFF
20 DIM OB (4000), ST (4300), DX(4100), T(20), NA (20), f(20), ZA(20)
30 REM ***********************************************************************
40 REM * This computer program designed to simulate a process by *
50 REM * using the VSI control charts of two sampling interval, *
60 REM * where the shift is linearly increasing. and occur at a *
70 REM * random point of time is the future *
80 REM ***********************************************************************
90 REM
100 RABNDOMIZE (TIMER)
110 REM
120 REM * REM *
130 REM * h=the width of the control limits. *
140 REM * h1=The width of the sectional limit area. *
150 REM * Slope = The shift increasing per unit time per process *
160 REM * Standard deviation *
170 REM * Delta0 =The shift value at time 0 *
180 REM * Ob () =The shift value at time 0 *
190 REM * St () =The sampling time of sample number (). *
200 REM * d1,d2=The minimum and maximum sampling intervals *
210 REM * p1,p2=The sampling intervals probabilities *
220 REM ***********************************************************************
230 REM
240 REM
250 REM
260 REM ***********************************************************************
270 REM * Entering the chart parameters *
280 REM ***********************************************************************
290 CLS: LOCATE 8,20: INPUT "Enter the width of the control limits"; H
300 LOCATE 9,20: input "Enter the sample size ":N
LOCATE 10,20 INPUT "Enter the fixed sampling interval"; FSI
LOCATE 11,20:INPUT "Enter the min. sampling interval"; D1
LOCATE 12,20:INPUT "Enter the max. sampling interval"; D2
LOCATE 13,20: INPUT "Enter the interval of first sample"; FD
LOCATE 14,20:INPUT "Enter the average of shift occurrence"; LAM
LOCATE 15,20:INPUT "Enter how many times to run"
REM *********************************************************
CLS; LOCATE 11,27:PRINT "The shift pattern menu"
LOCATE 14,23:PRINT "----------------------------------------"
LOCATE 14,23:PRINT "1) Exponentially Distributed"
LOCATE 14,23:PRINT "2) Normally Distributed"
LOCATE 16,23:PRINT "----------------------------------------"
LOCATE 17,23:INPUT "Enter your choice 1, or 2 " ; CH
CLS; ON CH GOTO 450,470
LOCATE 10,19:INPUT "The average of the shift exponential distribution";
LAMS
GOTO 490
LOCATE 10,19:INPUT "The mean of the shift normal distribution"; MSH
LOCATE 11,19:INPUT "The standard deviation of the normal distribution";
SGMAS
REM
REM
REM *****************************************************************
REM * Calculation of q value at the specified width of control *
REM * limits. Using formula 26.2.17 in Abramowitz and Stegun *
REM * (1965), Handbook of Mathematical Functions, Dover *
REM * Publication: Inc., NY *
REM *****************************************************************
A1=.31938153#
A2=-.356563782#
A3=1.781477937#
A4=-1.821255978#
A5=1.330274429#
REM * Entering the two variable sampling intervals and calculation
REM * their probabilities, such that the VSI is matched for FSI at
REM * the state of the statistical control

IF D1 = D2 THEN 800 ELSE P1=((D2-FSI)/(D2-1))*(1-Q);P2=1-Q-P1;GOTO 810
P1=(1-Q)/2;P2P1

REM * COMPUTING number of sigma for the sectional limit area
REM * using the inverse normal CDF formula 26.2.23 in Abramowitz and Stegun
REM * (1965). Handbook of Mathematical Functions, Dover Publications,
REM * Publications, Inc., NY.

P=1-(P1/2)-(Q/2);PROB=P
IF P>=1 THEN 800 ELSE 820
Analyzing the Performance of Variable Sampling Interval Control Charts

930   H1=1
940   GOTO 1100
950   IF P<=0 THEN 960 ELSE 980
960   H1=-1
970   GOTO 1100
980   IF P>.5 THEN P=1-P
990   G=1/(P^2)
1000  G=LOG(G)
1010  G=SQR(G)
1020  XP=1+1.432786*G+.189269*(G^2)+.0010308*(G^3)
1030  XP=(2.515517+,.802853*G+.010328*(G^2))/XP
1040  XP=XP-G
1050  IF PROB>.5 THEN XP=-XP
1060  H1=XP
1060  REM *****************************************************************************
1070  REM * Calculation of the control limits *
1080  REM *****************************************************************************
1090  MEAN=0;SIGMA=1;SIGMAX=SIGMA/(SQR(N))
1100  UCL=MEAN+H1*SIGMAX
1110  CL=MEAN
1120  LCL=MEAN-H1*SIGMAX
1130  USLA=MEAN+H1*SIGMAX
1140  LSLA=MEAN-H1*SIGMAX
1150  REM *****************************************************************************
1160  REM
1170  REM
1180  REM
1190  REM
1200  REM *****************************************************************************
1210  REM * Printing the output of chart parameters entered before *
1220  REM *****************************************************************************
1230  CLS
1240  PRINT " Simulation Inputs : using two (SVSI) control chart "

-439-
PRINT "Matched for FSI control chart, the shift is random variable "
PRINT "and occurs as an exponential random variable in the future"
PRINT " IF AE=1 THEN 1280 ELSE 1290"
PRINT " The two sampling intervals is symmetric around ";FSI," hour";GOTO 1300
PRINT "The two sampling intervals is asymmetric around the ";FSI," hour"
PRINT "The width of control limits is";H," process sigma"
PRINT "The width of sectional limits area";H1," process sigma"
PRINT " The sample size is"; N
PRINT " The fixed sampling interval is"; FSI," hour"
PRINT "The upper control limit is"; UCL,
PRINT "The lower control limits is"; LCL
PRINT "The upper sectional limit area is"; USLA
PRINT "The lower sectional limit area is"; LSLA
PRINT "The center line is"; CL
PRINT "The minimum sampling interval is"; D1," with" ;P1," probability"
PRINT "The maximum sampling interval is"; D2," with" ;P2," probability"
PRINT "The average time of shift occurrence is"; LAM
IF CH=1 THEN 1430 ELSE 1440
PRINT "The average of shift distribution is"; LAMS; GOTO 1460
PRINT "The average of shift distribution is"; MEANSH
PRINT "The standard deviation of shift distribution is"; SGMAS
LOCATE 20,1;INPUT "Press enter to continue ";DS$ 
IF DS$="" THEN 1320 ELSE 1100
REM ***********************************************
REM * STARTING THE MONITORING PROCESS *
REM ***********************************************
FOR LOOP1=1 TO 15
FOR LOOP =1 TO CIR
K1=RND
TS0=(-LAM)*LOG(1-K1)
I=1
Analyzing the Performance of Variable Sampling Interval Control Charts

1560 IF I=1 THEN 1570 ELSE 1580
1570 ST(I)=FD
1580 IF ST(I)<TS0 THEN 1590 ELSE 1610
1590 D(I)=0
1600 GOTO 1660
1610 REM ************************ Shift Random variable Generation ************************
1620 IF CH=1 Then 1630 ELSE 1640
1630 F1=RND:D(I)=(LAMS)*LOG(1-F1):GOTO 1660
1640 F2=RND:F3=RND:F4=(-2*1.0G(F2))^0.5*SIN(2*3.14*F3)
1650 REM ********************************************
1660 MEAN1=MEAN+d(I)*SIGMA
1670 K2=RND:K3=RND
1680 Z=(-2*LOG(K2))^0.5*SSIN(2*3.14*K3)
1690 IF D(I)=0 THEN 1700 ELSE 1710
1700 OB(I)=MEAN0+Z*SIGMAX:GOTO 1720
1710 OB (I)=MEAN1+Z*SIGMAX
1720 IF ST(I)<TS0 THEN J=J+1 ELSE J=0
1730 IF J=1 THEN O=ST(I)-TS0 ELSE 1750
1740 IF OB(I)>=UCL OR OB(I)<=LCL THEN 1790 ELSE 1750
1750 IF OB(I)<UCL AND OB(I)>=USLA THEN 1770 ELSE 1760
1760 IF OB(I)>LCL AND OB(I)<=LSLA THEN 1770 ELSE 1780
1770 I=I+1:ST(I)=ST(I-1)+D1:GOTO 1560
1780 I=I+1:ST(I)=ST(I-1)+D2:GOTO 1560
1790 REM ********************************************
1800 IF ST(I)<TS0 THEN 1770 ELSE 1810
1810 TIME=ST(I)-TS0
1820 ATS=ST(I)-TS0-O:CLS
1830 LOCATE 10,1:PRINT "GROUP#"; LOOP1."RUN#";1,LOOP
1840 PRINT "The adjusted time to signal is"; ST(I)-TS0
1850 PRINT "The number of samples taken after the shift occur are"; J
1860 PRINT "The time of shift starting is"; TS0
1870 PRINT "The chart signal at shift value equall"; D(I)
PRINT "The time between the shift starting point and next sample is"; 0
PRINT "The time from the first after the shift, sample until the shift is"; ATS
AVER=AVER+ST(I) - TS0
SAMPLE=SAMPLE+I; FTRA=FTRA+0; AT+ATS

I=0; NEXT
LOOP: AVER=AVER/CIR; SAMPLE=SAMPLE/CIR; FTRA=FTRA/CIR; AT=A T/CIR
AAT=AAT+AT; AATS=AATS+AVER; NSS=NSS+SAMPLE; AFTRA=AFTRA+AFTR A+FATRA: T(LOOP1)=AVER; NA(LOOP1)=SAMPLE; (LOOP1)=FATRA; ZA
(LOOP1)=AT; AVER=0; SAMPLE=0; FATRA=0; AT0
NXT LOOP: CLS; PLAY"d2"
PRINT" Group AAT ANSS ZAVR."
PRINT"-----------------------------------------------------------------------------"
FOR I=1 TO 15
PRINT I,T(I),NA(I),ZA(I)
NEXT I
PRINT"-----------------------------------------------------------------------------"
PRINT "Adjusted average time to signal is"; AATS/15
PRINT "Average number of samples taken after the shift are"; NSS/15
PRINT "Average time between shift point and next sample is"; AFTRA/15
PRINT "Average time to Starting from first sample after the shift is"; AAT/15