PROPORTIONAL REASONING AMONG
INTERMEDIATE, SECONDARY, AND COLLEGE
STUDENTS IN JORDAN

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Abstract

The concept of proportionality was selected for study in this investigation because it is crucial to the understanding of science. The study was designed to determine developmental patterns in proportional reasoning among Intermediate, Secondary, and College Students in Jordan.

Form A of the Test of Logical Thinking (TOLT) was administered to 269 students in three educational level. The students were classified into concrete, transitional, and formal levels on the basis of their scores on TOLT. Proportional reasoning was measured by the two items requiring proportional reasoning. A correct solution to an item required that a correct response and a correct reason are selected. All possible response combinations were analyzed.

The results of the study indicate that: (a) large percentages of students in various educational levels fail to solve problems requiring proportional reasoning; (b) the lower the educational level, the larger the percentage of students who could not solve problems; (c) students have more difficulty in handling problems with fractions than those involving whole numbers; (d) wrong responses seem to have a certain incorrect pattern of reasoning; and (e) proportional reasoning is a gradual and not an abrupt process. The implications of the results to science teaching are discussed in the paper.

Background

Formal thought has been the subject of many investigations by educational researchers in many countries around the world (Karplus et al., 1977; Karplus and Peterson, 1970; Levine and Linn, 1977; Lovell and Butterworth, 1966). The formation of formal reasoning patterns occurs mostly during the years of adolescence and has, therefore, great relevance to the teaching of science and mathematics at the secondary and college levels. The importance of cognitive development in learning science has been emphasized by many researchers and science educators. (Cantu and Herron, 1978; Goodstein and Howe, 1978). Several studies have indicated that different levels of science concepts require different levels of cognitive development. Goodstein and Howe (1978) suggested that formal thought is required in learning many science concepts taught in Junior high schools, senior high schools, and college science courses. Karplus et al (1977), after analyzing several science texts, argued that certain formal reasoning patterns, such as proportional reasoning and control of variables, are vital for the understanding of secondary school science.
Inhelder and Piaget (1978) described the growth of logical thought from age 8 to age fifteen. During this period children move from concrete reasoning to formal reasoning. At the concrete level the child reasons only about the specific or actual content of the problem. He can organize observed data using such processes as classification, seriation, and matching elements. These processes maintain information as it was in the original form. The concrete child also begins to recognize physical properties of the problem, consider different possibilities, and use a system of propositional logic. He is capable of generating many possible experiments or determine the relationships among variables.

According to Piaget the formal stage starts at about age 12 and the individual becomes formal at about age 15. However some experimental evidence indicates that there are individual differences among students of the same age group, school and background (Herron, 1975; Lawson and Renner, 1975; McKinnon and Renner, 1971). According to these researchers formal students will attain abstract concepts better than concrete students.

Of the various reasoning modes identified by researchers, proportional reasoning received greatest attention in the literature (Newton and Capie, 1981; Karplus et al., 1977; Levine and Linn, 1977; Wollman and Lawson, 1978). This mode of reasoning is considered essential for teaching any introductory science course. Many physical and chemical concepts such as density, gravitational force, and acceleration are names given to proportional relationships. Biology courses also involve proportions such as concepts of energetics. Students’ ability to comprehend and use proportions is therefore of major concern to science educators. Piaget considers proportionality schema as a primary acquisition at the stage of formal operations. Unfortunately recent evidence suggests that as much as 50% of some samples of secondary school and college-age students in several countries (such as U.S.A., Denmark, U.K., Sweden, Italy, Austria and Germany) have failed to acquire understanding of proportionality schema (Karplus and Peterson, 1970; Lawson and Renner, 1974; Lunzer, 1965; Wollman and Karplus, 1974; Thornton and Fuller, 1981; Karplus, et al., 1977).

Lawson, et al. (1984) tested the hypothesis that successful performance on specific proportions problems, such as the pouring water task used by Lawson, Karplus and Adi (1978), is dependent upon the prior acquisition of general hypothetico-deductive reasoning abilities. This general ability manifests itself as the student asks himself the question: is the answer I have selected correct? As an initial test of the hypothesis, students were classified into additive, transitional, or proportional reasoning categories based upon responses to a proportions task. The researcher reported that proportional students performed better than transitional students who in turn performed better that additive students on a number of items involving hypothetico-deductive reasoning abilities.
Thornton and Fuller (1981) administered problems which could be solved using proportional reasoning to 8000 college students throughout the United States. Students responses were categorized into: (i) intuitive (a guess with little evidence of reasoning); (ii) additive (adds or substracts to obtain an answer); (iii) ratio attempt (attempts a ratio but fails for reasons other than arithmetic); (iv) ratio formula (uses proportional reasoning). They concluded that: (i) college students use a variety of problem solving approaches to proportions problems; (ii) college teachers ought not to assume that even obvious ratio problems will provoke all students to use proportional reasoning; and (iii) additive reasoning is used by many students in inappropriate situations.

This emphasis in the research on proportional reasoning underscores the important role that proportional reasoning has in the attainment of concepts. This study resulted from a desire to explore the development of proportional reasoning.

The Study

Since the proportionality schema is crucial to the understanding of science concepts (and to intellectual development in general) it was selected for study in this investigation. The study, therefore, is designed to determine developmental patterns in proportional reasoning among intermediate school, senior secondary school, and college level students in Jordan. Analysis of incorrect response patterns may help in understanding the development of proportional reasoning in adolescents.

More specifically, the study would attempt to answer the following questions:

1. What proportions of intermediate school, secondary school, and college students utilize proportional reasoning to solve simple proportional reasoning problems?

2. What are the most commonly observed response patterns for each proportional reasoning problem?

3. To what extent do students utilize additive reasoning to solve proportional reasoning problems?

4. Is the development of proportional reasoning a gradual or abrupt process?

Methods

The subjects: Form A of the Test of logical Thinking (TOLT), (Tobin and Capic, 1980) was translated into Arabic by the researcher, and was administered to 269 students in three educational levels: 78 Intermediate students (grade 9, age 14-15); 89 secondary students (grade 11, age 16-17), and 102 college level students (First year, age 18-19). One class from each of two intermediate schools, two secondary schools, a University, and a Community College was randomly
selected to form the sample for the study. The sample at each level included both male and female students.

The Test: TOLT was developed by modifying Lawsons' Test, a previously constructed measure of formal reasoning ability (Tobin and Capie, 1980). Items that had been used in prior research (Lawson, 1978) were used as a basis for developing an initial version of TOLT. "The adoption of this procedure assured that TOLT would contain items that had been previously reported as valid measures of formal reasoning ability" (Tobin and Capie, 1980, p. 3) The test was modified so that multiple justification as well as multiple solutions were provided for each item. A correct solution to an item required that the correct response and the correct justification (reason) for that response were selected. Since each had five responses (choices) and five reasons (justifications), there were 25 possible combinations for each question.

The test consists of 10 items designed to measure five modes of format reasoning with two items for each mode: controlling variables; proportional reasoning; probabilistic reasoning; correlational reasoning; and combinatorial reasoning. The mode of reasoning represented by each item is indicated by the face validity of the items and by the validating procedures adopted by Tobin and Capie (1980) and Lawson (1978). (See Appendix).

Data reported by Tobin and Capie (1980) suggested that the 10 items were measuring a common underlying dimension. "Evidence of this was a high internal consistency reliability (alpha = 0.85) and a comparatively strong one factor solution obtained from factor analysis of the achievement data on each of the reasoning modes". (Tobin and Capie, 1980, p.4).

An administration of the translated version of TOLT to a sample of 212 eleventh grade students in Jordan revealed an alpha coefficient of internal consistency of 0.70 (Ahlawt and Billeh, in press). Moreover, Tobin and Capie (1980) concluded that TOLT provides a convenient means of obtaining valid and reliable measures of formal reasoning ability for researchers and teachers, particularly when data are required for groups of subjects.

Proportional reasoning was measured by the two items requiring proportional reasoning (see Appendix). They involve making Orange juice form Oranges. The ratio used in the two problems is 3:2. In the first question, the students were expected to obtain the answer by "9" glasses given "6" Oranges. In the second question they were expected to obtain the answer "8 2/3" Oranges given "13" glasses.

Data Analysis: The Arabic version of the test was administered to 269 students from the three educational levels mentioned above. The 10 items were scored to obtain a reasoning score for each student. Students were classified into three groups representing three developmental stages at each educational level.
using TOLT scores 0-10 as follows: Concrete (0-3); Transitional (4-6); and Formal (7-10). The students were also grouped by educational level to Intermediate (I), Secondary (S) and College (C) level. To test the significance between percentages of each group of students answering each item correctly, the z test was utilized (alpha = 0.01). To test the relationship between the educational level and the number of students answering proportional items correctly, the Chi Square test was utilized and the coefficient of contingency of each item was computed and tested for significance (alpha = 0.01).

Responses on the two items measuring proportional reasoning were recorded and the 25 possible combinations of responses and justification of responses were noted. The frequencies of the three-four most common combinations were determined for each set of data and were selected for further analysis. Graphs of percentages of subjects selecting a response against subjects sorted by TOLT scores 0-10 were plotted for each of the two items.

The Results

An examination of Table 1 revealed that only 10.2% of the intermediate students answered the first item correctly, while the corresponding percentages for secondary and college students were 42.7% and 63.7%. When the z test for significance between percentages was utilized, the percentages were found to be statistically different from each other at p < 0.01 (z values were 4.7, 2.9, and 7.3 respectively). The percentages for the second item (for which the answer was a fraction) were consistently lower than those for the first item (for which the answer was a whole number). These percentages were 6.4%, 32.6% and 43.1% for intermediate, secondary, and college students respectively (Table 1).

Table 1: Percentage of students correctly answering proportional items by educational level.

<table>
<thead>
<tr>
<th>Educational Level</th>
<th>I</th>
<th>S</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item One</td>
<td>10.2</td>
<td>42.7</td>
<td>63.7</td>
</tr>
<tr>
<td>Item Two</td>
<td>6.4</td>
<td>32.6</td>
<td>43.1</td>
</tr>
</tbody>
</table>

I = Intermediate; S = Secondary, C = College

When the z test was utilized the percentage for intermediate students was found to be statistically different from that for secondary and college students at p < 0.01 (z values were 4.2 and 5.5 respectively). The percentage for secondary students was not found to be statistically different than that for college students (z = 1.5). When the percentages for the first and second items were compared for each group of students, the resulting z values (0.95, 1.4, and 2.9 for intermediate, secondary,
and college students respectively) indicated that there were significant differences (at $p < 0.01$) between the percentages for college students only.

To determine the relationship between the educational level and the number of students answering each proportional item correctly, the Chi Square test was utilized and the coefficient of contingency was computed for each item. The values for Chi Square were 57.74 and 29.72 for the first and second items respectively. The corresponding values for the coefficients of contingency were 0.42 and 0.32 respectively. Both values were found to be statistically significant at $p < 0.0001$.

These results indicate that: (a) large numbers of students at various educational levels failed to solve problems requiring proportional reasoning; (b) the lower the educational level, the larger the percentage of students who could not solve problems; and (c) college students have more difficulty in handling problems with fractions than in those involving whole numbers.

When the 25 response combinations for the first proportional item were examined, the following three most common response combinations were revealed, in order of frequency:

1. Proportional response (correct answer and correct reason).
2. Additive reasoning (focus on a single difference).
3. No way to predict.

Similarly an examination of the 25 possible response combinations for the second proportional item revealed the following four common response combinations, in order of frequency:

1. Proportional response.
2. Additive reasoning.
3. No way to predict.
4. Correct reason with arithmetically wrong answer.

Table 2 shows the percentages of students selecting common responses by educational level within developmental level for the first and second items. These results indicate that

a. almost all formal students selected correct responses and correct reasons;
b. larger percentages of transitional students selected the correct combination than concrete students. When the percentages for the transitional and concrete students at each educational level were compared to determine if significant differences exist, the resulting $z$ values were all significant at $p < 0.01$. ($z$ values for intermediate, secondary, and college students respectively were 4.7, 3.2, and 3.8 for item 1; and 6.3, 2.8, and 5.3 for item 2).

The most common incorrect response was that of additive thinking which involves a focus on one difference by the students. In the first item additive students
**Table 2:** Percentage of students selecting a common response by educational level within developmental level.

<table>
<thead>
<tr>
<th>Item One</th>
<th>Concrete</th>
<th>Transitional</th>
<th>Formal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I  S  C</td>
<td>I  S  C</td>
<td></td>
</tr>
<tr>
<td>Proportional</td>
<td>3 33 50</td>
<td>50 74 88</td>
<td>50 100 100</td>
</tr>
<tr>
<td>Additive</td>
<td>69 30 81</td>
<td>25 11 9</td>
<td>0 0 0</td>
</tr>
<tr>
<td>No way to Predict</td>
<td>7 16 15</td>
<td>13 5 0</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Item Two</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I  S  C</td>
<td>I  S  C</td>
<td></td>
</tr>
<tr>
<td>Proportional</td>
<td>0 22 24</td>
<td>50 53 81</td>
<td>100 100 100</td>
</tr>
<tr>
<td>Additive</td>
<td>50 31 27</td>
<td>0 16 16</td>
<td>0 0 0</td>
</tr>
<tr>
<td>No way to predict</td>
<td>13 8 10</td>
<td>0 5 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>Correct reason arithmetically wrong answer</td>
<td>4 11 6</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>

selected the choice "8" and followed that by the reason: "with 4 oranges the difference was 2; with 6 oranges, the difference would be 2 more". This response was more common among concrete students than among transitional or formal students. And within the concrete reasoning group it was more common among intermediate students than among secondary or college students.

The second most common wrong response was that of: "there is no way to predict". This response was selected mostly by concrete students, most of whom were secondary and college students. The reason for this may by attributed to the fact that secondary and college students are attracted to such a response because they are more acquainted with multiple choice items than intermediate students, and tend to think that a certain trick is involved in the item.

The third most common incorrect response (on the second item) was that of choosing a mathematically wrong answer with a correct reason, particularly by concrete students, perhaps because they possess poorer arithmetic skills than transitional or formal students.

To answer the question whether proportional reasoning is a gradual or abrupt process, plots of percentages of subjects selecting a response against subjects sorted by TOLT scores 0-10 were made for each of the two items (Figures 1 and 2).

These Figures seem to suggest that proportional reasoning pattern is gradual and not abrupt (with the exception of a reduced percentage at a score of 6 and 5 for the first and second items, respectively). The chance of selecting the correct response increases with an increase of the TOLT score. Similarly, additive reasoning is used
Fig. 1. Percent of subjects selecting a response against subjects sorted by TOLT scores more among subjects with low TOLT scores. Other incorrect responses seem to be more attractive to students with low TOLT scores.

Discussion and Implications

The results of the present study provide further evidence to observations by other researchers (Karpus et al., 1977; and Newton and Tobin, 1981) that large numbers of adolescents have not attained proportional reasoning.

On the basis of the research reported here, some conclusions can be drawn. Intermediate, secondary and college teachers should not assume that the majority of their students utilize proportional reasoning in solving ratio problems. Many students at the intermediate and secondary level still utilize additive reasoning. Thurston and Fuller (1981) arrived at similar conclusions and suggested that “if this does indicate a lack of formal reasoning ability, then the Piagetian level of the students must be considered in planning science courses”. Karplus et al (1977) conducted a survey of proportional reasoning and control of variables among 13-to-15-year-old students in seven countries (Denmark, Sweden, Italy, U.S.A., Austria, U.K. and Germany) and concluded “that a substantial fraction of students lack the
ability to articulate proportional reasoning. Science and mathematics programs in all but top level of selective schools should take this diversity of student reasoning into account in so far as content selection, laboratory activities, and textbook choices are concerned".

![Graph showing percentage of subjects selecting a response against subjects sorted by TOLT scores.]

Fig. 2 Percent of subjects selecting a response against subjects sorted by TOLT scores.

Knowledge of the teachers that concrete and transitional students are still additive reasoners helps the teachers to develop certain methods to present subject matter to different groups of students. Wollman and Lawson (1978) suggested that in teaching additive students "problems should initially involve small whole numbers and only gradually include large nonintuitive relationships. Algorithmic solutions should not be introduced until after the student has the opportunity to apply his own conceptual resources to the problems".

With regard to the question of whether the development of proportional reasoning is a gradual or abrupt phenomenon, the results seem to suggest that it is
gradual. These results provide support to a similar conclusion arrived at by Pailrand (1979) when studying the same three educational levels as used in this study. Although this research question would ideally involve a different design, the results in this study do seem to provide further support to the idea of continuous development. Further studies are needed to probe more deeply into the development of formal reasoning abilities by concrete, transitional, and formal thinkers.

Because the concepts of proportionality is essential in learning science, mathematics, and other subject matter involving proportions, and because it is lacking in a large percentage of students at various educational levels, several researchers tried to find out if teaching has any influence on the development of the concept. Karplus et al. (1977) concluded that teaching can have some influence on the development of reasoning by the students in the age range 13-15, and that the development of reasoning patterns should be an important objective of teaching programs for 13-15-year-olds.

Wollam and Lawson (1978), in an effort to develop a strategy for teaching the concept of proportionality in seventh graders, stated that: "Instruction designed to improve reasoning should, as far as possible, parallel the process of internalization of actions. This would involve working first with the principle (in this case proportions) in concrete, flexible, and action-oriented contexts, manipulating materials and actually "seeing" the principle operate. The students can be supplied with symbolic representations \( \frac{a}{b} = \frac{c}{d} \) of the principle so that it can be internalized by gradually reducting the need for pereceptual and motor support".

To explain the diversity of students' responses and the inability of some college and secondary school students to solve proportionality problems, Piaget hypothesized that persons who were otherwise formal operational may fail to use formal reasoning in specific task situations because they lack familiarity with the phenomenon in question (Lawson et al. 1984). Therefore familiarization of students with different problem formats and development of instrucational strategies for teaching proportionality to intermediate, and college students seem to be useful tools for teachers who wish to assist their students in developing proportionality, and other reasoning patterns, and enable them to acquire science, mathematics, and other concepts which otherwise may not be acquired by these students.
PROPORTIONAL REASONING

TEST OF LOGICAL THINKING
Sample Items

**Item One** (Proportional reasoning).

Four large oranges are squeezed to make six glasses of juice. How much juice can be made from six oranges?

a. 7 glasses  
b. 8 glasses  
c. 9 glasses  
d. 10 glasses

**Reason**

1. The number of glasses compared to the number of oranges will always be in the ratio 3 to 2.
2. With more oranges, the difference will be less.
3. The difference in the numbers will always be two.
4. With four oranges the difference was 2. With six oranges the difference would be two more.
5. There is no way of predicting.

**Item Two** (Proportional reasoning)

How many oranges are needed to make 13 glasses of juice?

a. 6 1/2 oranges  
b. 8 2/3 oranges  
c. 9 oranges  
d. 11 oranges  
e. other

**Reasons**

1. The number of oranges compared to the number of glasses will always be in the ratio 2 to 3.
2. If there are seven more glasses, then five more oranges are needed.
3. The difference in the numbers will be two.
4. The number of oranges will be half the number of glasses.
5. There is no way of predicting the number of oranges.

**Item 5** (Probabilistic reasoning)

A gardener bought a package containing 3 squash seeds and 3 bean seeds. If just one seed is selected from the package what are the chances that it is a bean seed?
a. 1 out of 2
b. 1 out of 3
c. 1 out of 4
d. 1 out of 6
e. 1 out of 6

Reasons
1 - Four selections are needed because the three squash seeds could have been chosen in a row.
2 - There are six seeds from which one bean seed must be chosen.
3 - One bean seed needs to be selected from a total of three.
4 - One half of the seeds are bean seeds.
5 - In addition to a bean seed, three squash seeds could be selected from a total of six.

Item 6 (Probabilistic reasoning)

A gardener bought a package of 21 mixed seeds. The package contents listed:

3 short red flowers
4 short yellow flowers
5 short orange flowers
4 tall red flowers
2 tall yellow flowers
3 tall orange flowers

If just one seed is planted, what are the chances that the plant that grows will have red flowers?

a. 1 out of 2
b. 1 out of 3
c. 1 out of 7
d. 1 out of 21
e. other

Reason
1 - One seed has to be chosen from among those that grow red, yellow or orange flowers.
2 - 1/4 of the short and 4/9 of the tall are red.
3 - It does not matter whether a tall or a short is picked. One red seed needs to be picked from a total of seven red seeds.
4 - One red seed must be selected from a total of 21 seeds.
5 - Seven of the twenty one seeds will produce red flowers.
References


Thurston, Melvin and Fuller, Robert, "How do college students solve proportion problems?" *Journal of Research in Science Teaching, 1981, 18*: 335-340
